Collateral Requirements and Asset Prices*

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January 12, 2011

Abstract

In this paper we examine the effect of collateral on the prices of long-lived assets such as stocks or housing. We consider a Lucas style exchange economy with heterogenous agents and collateral constraints. There are two trees in the economy which can be used as collateral for short-term loans. For the first tree the collateral requirement is determined endogenously as in Geanakoplos and Zame (2003) while the collateral requirement for loans on the second tree are exogenously regulated. We show that the presence of collateral constraints and the endogenous margin requirements for the first tree lead to large excess volatility of the second tree and that changes in the regulated margin requirements for the second tree have large effects on the volatility of both trees, where the effect on the second tree can be of the opposite sign as the effect on the regulated tree. In our calibration we follow Barro and Jin (2009) and allow for the possibility of disaster states. This leads to very large quantitative effects of collateral requirements and to realistic average excess returns of the second tree. We show that our qualitative results are robust to the actual parameterization of the economy.

*Preliminary and incomplete.

1 Introduction

Many previous papers have formalized the idea that borrowing on collateral might give rise to cyclical fluctuations in real activity and enhance volatility of prices (see e.g. Geanakoplos (1997), Kiyotaki and Moore (1997) and Aiyagari and Gertler (1999)). In these models, it is possible to have substantial departures of the market price from the corresponding price under frictionless markets. These results have led researchers to suggest that by managing leverage (or the amount of collateralized borrowing), a central bank can reduce aggregate fluctuations (see e.g. Ashcroft et al (2010) or Geanakoplos (2010)). However, establishing the quantitative importance of the financial multiplier that drives these excess volatility results has been a challenge in the literature (see Kocherlakota (2000) or Cordoba and Ripoll (2004)). Moreover, so far, there have been few quantitative studies that take into account that a household can use several different assets as collateral, and that regulated margin requirements for loans on one asset might have important effects on the volatility of other assets in the economy.

In this paper, we examine a Lucas (1978) style exchange economy with heterogeneous agents and assume that households can only take short positions if they hold a infinitely lived asset (a tree) as a long position. This model was first analyzed by Kubler and Schmedders (2003) and subsequently used by Cao (2009). As in Kubler and Schmedders (2003) and Geanakoplos and Zame (2002) we assume that agents can default on a negative bond position at any time without any utility penalties or loss of reputation. As in Geanakoplos and Zame (2002) margin requirements are endogenously determined by market forces. However, we introduce a real cost of default to the lender which will make it relatively unattractive for agents to trade in bonds with low collateral requirements. We assume that there are two types of agents with Epstein-Zin (1989) (recursive) utility who distinguish themselves by their risk-aversion. The agent with the low risk-aversion is the natural buyer of risky assets and when the economy is hit with a negative shock his collateral constraint forces him to sell of the stocks to the more risk-averse agent which potentially leads to a substantial drop in prices. We show under which conditions on preferences, default costs and aggregate shocks this effect is quantitatively significant and explain how a regulation of margin requirements can decrease price volatility. Our main results are about a model with two trees. The tree's have identical payouts but distinguish themselves by the way in which they may be used as collateral. We assume that the first tree can be used as collateral with endogenous margin requirements, while the second tree can only be used as collateral with regulated margin requirements. We show that first and second moments of the trees' returns vary hugely with the margin requirement on the second tree. In particular .....

Lucas (1978) considers an exchange economy with a single perishable consumption good where infinitely lived identical agents trade shares in physical assets ("trees"). We extend the model to allow for heterogeneous agents who not only can trade in trees but can also make promises by selling financial securities. As in the two-period model of Geanakoplos and Zame (2000), agents can default on these promises at any time without any utility penalties or loss of reputation. Financial securities are therefore only traded if the promises associated with these securities are backed by collateral. The only collateral in this economy consists of shares in the physical assets.

The vast majority of debt, especially if it extends over a long period of time, is guaranteed by tangible assets called collateral. For example, residential homes serve as collateral for short- and long-term loans to households, and investors can borrow money to establish a position in stocks, using these as collateral. The margin requirement dictates how much collateral one has to hold in order to borrow one dollar. Depending on the asset that is used as a collateral, market forces might play an important role in establishing these collateral requirements. For stocks the situation is not obvious: The Federal Reserve Board sets minimum margin requirements for broker-dealer loans, using something called Regulation T. In fact, until 1974, the Fed considered initial margin percentages as an active component of monetary policy and changed them fairly often (see Willen and Kubler (2006)). In the US housing market, there are no such regulations and margins can be arbitrarily small. Following Geanakoplos (1997) and Geanakoplos and Zame (2002), we endogenize the margin requirements by introducing a menu of financial securities. All securities promise the same payoff, but they distinguish themselves by the margin requirement. In equilibrium only some of them will be traded, thereby determining an endogenous margin requirement. This implies of course, that for many bonds and many next period's shocks, the face value of the debt falls below the value of the collateral. Following Geanakoplos and Zame (2002) we make the strong assumption that an agent can default on individual promises without declaring personal bankruptcy and giving up all the assets he owns. There are no penalties for default and a borrower will always default once the value of the debt is above the value of the collateral. Since this implies that the decision to default on a promise is independent of the debtor, we do not need to consider pooling of contracts as in Dubey et al. (2000), even though there may be default in equilibrium. This treatment of default is somewhat unconvincing since default does not affect a household's ability to borrow in the future and it does not lead to any direct reduction in consumption at the time of default. Moreover, declaring personal bankruptcy typically results in a loss of all assets, and it is rarely possible to default on some loans while keeping the collateral for others. However, there do exist laws for collateralizable borrowing where default is possible without declaring bankruptcy. Examples include pawn shops and the housing market in many US states, in which households are allowed to default on their mortgages without defaulting on other debt. It is certainly true that the recent 2008 housing crises makes this assumption look much better.

To obtain a better understanding of the impact of collateral constraints and default on the dynamic behavior of economies with incomplete markets, it is necessary to compute equilibria in such models. There are no analytical results for the dynamic economies we consider. We use the theoretical results in Kubler and Schmedders (2003) and the computational algorithm from Brumm and Grill (2010) to solve the model numerically.

We assume that there are two agents with Epstein-Zin utility and a CES-aggregator. The agents have identical elasticities of substitution (IES) but distinguish themselves by their risk-aversion (RA). In the base-line model there is a single Lucas tree that can be used as collateral and we exogenously assume that collateral requirements are set to the lowest possible level that still ensures that there is never default in equilibrium. In order to obtain a sizable market price of risk, we follow the specification in Barro and Jin (2009) and introduce a 'disaster-shock' into the otherwise standard calibration. In this calibration, the effect of scarce collateral on the volatility of long-lived assets is quantitatively very large, at least when we assume an IES above 1. Exogenously tightening the margin requirements initially leads to an increase in price volatility, but at some point decreases volatility substantially.

We extent the basic model by endogenizing the collateral requirement as in Geanakoplos and Zame (2002) and Araujo et al (2010). We go beyond these papers in that we incorporate a cost of default. In our benchmark calibration, volatility decreases as the number of bonds increases if margin requirements are endogenous and there is no cost of default. However, realistic (small) costs of default shut down the markets for most bonds and at the end only the 'no-default'-bond is traded in equilibrium.

The main contribution of the paper is to consider a situation with two trees which have identical cash-flows but distinguish themselves by their 'collateralizability'. In the extreme case we assume that only one of the stocks can be used as collateral. In this case, we show that the volatility of the collateralizable tree is significantly smaller than in the baseline case while the volatility of the tree that cannot be used as collateral is much bigger. Average volatility is roughly the same. A natural interpretation of these findings is to identify the collateralizable tree with housing while the non-collaterlizable tree can be identified as the aggregate stock market. Using stocks as collateral is subject to many regulations and often very costly, while individuals can easily use houses. Volatility and excess returns for houses is much smaller than for stocks, which is in line with our findings.

We then consider the case where for one asset the margin-requirement is exogenously regulated (at a very high level) while for the other one margins are endogenous. We find that the regulation of the collaterizablity of the first asset has very large effects on the volatility of the second. In particular, in our calibration a tightening of margin requirements for the regulated tree uniformly decreases volatility of the unregulated one. For the regulated tree, tighter margins initially increase the price volatility and only decrease it for very large haircuts (above 75 percent).

The remainder of this paper is organized as follows. We introduce the model in Section 2. In Section 3 we discuss results with one tree. Section 4 focuses on the case of two trees. In Section 5 we consider extensions and sensitivity analysis and Section 6 concludes.

2 The Economic Model

We examine a model of an exchange economy that extends over an infinite time horizon and is populated by infinitely-lived heterogeneous agents.

The Physical Economy

Time is indexed by $t = 0, 1, 2, \ldots$. A time-homogeneous Markov chain of exogenous shocks (s_t) takes values in the finite set $\mathcal{S} = \{1, \ldots, S\}$. The $S \times S$ Markov transition matrix is denoted by π. We represent the evolution of time and shocks in the economy by a countably infinite event tree Σ. The root node of the tree represents the initial shock s_0. Each node of the tree, $\sigma \in \Sigma$, describes a finite history of shocks $\sigma = s^t = (s_0, s_1, \ldots, s_t)$ and is also called date-event. We use the symbols σ and s^t interchangeably. To indicate that $s^{t'}$ is a successor of s^t (or s^t itself) we write $s^{t'} \succeq s^t$. In a slight abuse of notation we use the notation σ_0^* to refer to initial conditions of the economy prior to $t = 0$.

At each date-event $\sigma \in \Sigma$ there is a single perishable consumption good. The economy is populated by H agents, $h \in \mathcal{H} = \{1, 2, \ldots, H\}$. Agent h receives an individual endowment in the consumption good, $e^h(\sigma) > 0$, at each node. In addition, at $t = 0$ the agent owns shares in Lucas trees. We interpret these Lucas trees to be physical assets such as firms, machines, land or houses. There are A different such assets, $a \in \mathcal{A} = \{1, 2, \ldots, A\}$. At the beginning of period 0, each agent h owns initial holdings $\theta_a^h(\sigma_0^*) \geq 0$ of tree a. We normalize aggregate holdings in each Lucas tree, that is, $\sum_{h \in \mathcal{H}} \theta_a^h(\sigma_0^*) = 1$ for all $a \in \mathcal{A}$. At date-event σ, we denote agent h's (end-of-period) holding of Lucas tree a by $\theta_a^h(\sigma)$.

The Lucas trees pay positive dividends $d_a(\sigma)$ in units of the consumption good at all date-events. We denote aggregate endowments in the economy by

$$\bar{e}(\sigma) = \sum_{h \in \mathcal{H}} e^h(\sigma) + \sum_{a \in \mathcal{A}} d_a(\sigma).$$

The agents have preferences over consumption streams representable by the following recursive utility function, see Epstein and Zin (1989, 1991),

$$U^h(c, s^t) = \left\{ \left[c^h(s^t)\right]^{\rho^h} + \beta \left[\sum_{s_{t+1}} \pi(s_{t+1}|s_t) \left(U^h(c, s^{t+1})\right)^{\alpha^h} \right]^{\frac{\rho^h}{\alpha^h}} \right\}^{\frac{1}{\rho^h}}, \tag{1}$$

where $\frac{1}{1-\rho^h}$ is the intertemporal elasticity of substitution (IES) and $1 - \alpha^h$ is the relative risk aversion of the agent.

Markets

At each date-event agents can engage is security trading. Agent h can buy $\theta_a^h(\sigma) \geq 0$

shares of tree a at node σ for a price $q_a(\sigma)$. Agents cannot assume short positions of the Lucas trees. Therefore, the agents make no promises of future payments when they trade shares of physical assets and thus there is no possibility of default.

In addition to the physical assets, there are J financial securities, $j = \mathcal{J} = \{1, 2, \ldots, J\}$, available for trade. These assets are one-period securities in zero-net supply. Security j traded at node s^t promises a payoff of one unit of the consumption good at each immediate successor node s^{t+1}. We denote agent h's (end-of-period) portfolio of financial securities at date-event σ by $\phi^h(\sigma) \in \mathbb{R}^J$ and denote the price of security j at this date-event by $p_j(\sigma)$. Whenever an agent enters a short position in a financial security j, $\phi^h_j(\sigma) < 0$, she promises a payment in the next period. In our economy such promises must be backed up by collateral holdings.

Agents can only sell (short) a financial security if they hold shares of trees as collateral. At each node σ, we associate an A-dimensional vector $k^j(\sigma) > 0$ of collateral requirements with each financial security $j \in \mathcal{J}$. If an agent sells one unit of security j, then she is required to hold $k^j_a(\sigma)$ units of each tree $a = 1, \ldots, A$ as collateral. If an asset a can be used as collateral for different financial securities, the agent is required to buy $k^j_a(\sigma)$ shares for each security $j = 1, \ldots, J$. (The 'collateral requirements' k^j_a may vary across date-events, see Section 2.1 below.) In the next period the agent can default on her earlier promise. In this case the agent loses the collateral she had to put up. In turn, the buyer of the financial security receives this collateral associated with the initial promise.

Since there are no penalties for default, a seller of security j at date-event s^{t-1} defaults on her promise at a successor node s^t whenever the initial promise exceeds the current value of the collateral, that is, whenever

$$1 > \sum_{a \in \mathcal{A}} k^j_a(s^{t-1}) \left(q_a(s^t) + d(s^t) \right).$$

The payment by a borrower of security j at node s^t is, therefore, always given by

$$f_j(s^t) = \min \left\{ 1, \sum_{a \in \mathcal{A}} k^j_a(s^{t-1}) \left(q_a(s^t) + d(s^t) \right) \right\}.$$

Our model includes the possibility of costly default. This feature of the model is meant to capture default costs such as legal cost or the physical deterioration of the collateral asset. For example, it is well known that housing properties in foreclosure deteriorate because of moral hazard, destruction, or simple neglect. We model such costs by assuming that part of the payment of the borrower is lost and thus the payment received by the lender is smaller than the borrower's payment. Specifically, the loss is proportional to the difference between the face value of the debt and the value of collateral, that is, the loss is $\lambda \left(1 - k^j_a(s^{t-1}) \left(q_a(s^t) + d_a(s^t) \right) \right)$ for some parameter $\lambda \geq 0$. The resulting payment to the lender of the loan in security j is thus given by

$$r_j(s^t) = \begin{cases} 1 & \text{if } k_a^j(s^{t-1})(q_a(s^t) + d_a(s^t)) \geq 1 \\ 0 & \text{if } (1+\lambda)k_a^j(s^{t-1})(q_a(s^t) + d_a(s^t)) - \lambda < 0 \\ (1+\lambda)k_a^j(s^{t-1})(q_a(s^t) + d_a(s^t)) - \lambda & \text{otherwise.} \end{cases}$$

This repayment function does not capture all costs associated with default. For example, it does not allow for fixed costs independent of the magnitude of the loss to the lender due to the default by the borrower. However, our functional form offers the advantage that the resulting model remains tractable since the repayment function is continuous in the value of the collateral.

Financial Markets Equilibrium with Collateral

For the definition of a financial markets equilibrium, it is helpful to define the terms $[\phi_j^h]^+ = \max(0, \phi_j^h)$ and $[\phi_j^h]^- = \min(0, \phi_j^h)$.

DEFINITION 1 *A financial markets equilibrium for an economy with initial tree holdings* $(\theta^h(\sigma_0^*))_{h\in\mathcal{H}}$ *and initial shock* s_0 *is a collection of agents' portfolio holdings and consumption allocations as well as security prices*

$$\left((\bar\theta^1(\sigma), \bar\phi^1(\sigma), \bar c^1(\sigma)), \ldots, (\bar\theta^H(\sigma), \bar\phi^H(\sigma), \bar c^H(\sigma)); \bar q_1(\sigma), \ldots, \bar q_A(\sigma), \bar p_1(\sigma), \ldots, \bar p_J(\sigma)\right)_{\sigma\in\Sigma}$$

satisfying the following conditions:

(1) Markets clear:

$$\sum_{h\in\mathcal{H}} \bar\theta^h(\sigma) = 1 \quad \textit{and} \quad \sum_{h\in\mathcal{H}} \bar\phi^h(\sigma) = 0 \quad \textit{for all } \sigma \in \Sigma.$$

(2) For each agent h*, the choices* $\left(\bar\theta^h(\sigma), \bar\phi^h(\sigma), \bar c^h(\sigma)\right)$ *solve the agent's utility maximization problem,*

$$\begin{aligned} \max_{\theta\geq 0,\phi,c\geq 0} U_h(c) \quad & \textit{s.t.} \quad \textit{for all } s^t \in \Sigma \\ c(s^t) \quad = \quad & e^h(s^t) + \sum_{j\in\mathcal{J}} \left([\phi_j(s^{t-1})]^+ r(s^t) + [\phi_j(s^{t-1})]^- f_j(s^t)\right) + \\ & \theta^h(s^{t-1}) \cdot \left(\bar q(s^t) + d(s^t)\right) - \theta^h(s^t) \cdot \bar q(s^t) - \phi^h(s^t) \cdot \bar p(s^t) \\ 0 \quad \leq \quad & \theta_a^h(s^t) + \sum_{j\in\mathcal{J}} k_a^j(s^t)[\phi_j^h(s^t)]^-, \quad a = 1, \ldots, A. \end{aligned}$$

The approach in Kubler and Schmedders (2003) can be used to prove existence. The only non-standard part, besides the assumption of recursive utility, which can be handled easily, is the assumption of default costs. Note however, that our specification of these costs still leaves us with a convex problem and standard arguments for continuity of best responses go through. To approximate equilibrium numerically, we use the algorithm in Brumm and Grill (2010). In Appendix A, we describe the computations and the numerical error analysis in detail.

2.1 Margins, haircuts and endogenous collateral requirements

The specification of $k_a^j(s^t)$ for bond j, trees a and across date-events s^t has important implications for equilibrium prices and allocations. We assume throughout the analysis of our model that for each bond $j \in \mathcal{J}$ only a particular tree can be used as collateral. For security j we denote by $a(j)$ the single Lucas tree that can be held as collateral to take a short-position in this security. We consider three different scenarios for the determination of $k_a^j(s^t)$.

One of the contributions of this paper is to endogenize collateral requirements. For this, we follow Geanakoplos (1997) and Geanakoplos and Zame (2002) who suggest a simple and tractable way to endogenize the collateral requirements by assuming that in principle any collateral requirement could be traded and to note that with scarce collateral only few of them will be traded in equilibrium. Following this approach, we denote the S direct successors of a node s^t by $(s^t, 1), ..., (s^t, S)$. We say that margin requirements for bonds which use as collateral some tree $\bar{a}$ are endogenous if there is a set of assets $\mathcal{J}_{\bar{a}}$ with $a(j) = \bar{a}$ for all $j \in \mathcal{J}_{\bar{a}}$ which has the property that for each shock next period s' there is precisely one bond which satisfies $k_{\bar{a}}^j(s^t)\left(q_a(s^t, s') + d(s^t, s')\right) = 1$. Generally this set will contain S assets but note that the one with the lowest collateral requirement is redundant in our framework, since it is collinear to the tree itself.

It is easy to see (as in Araujo et al. (2010)) that adding additional bonds with other collateral requirements (also only using tree $\bar{a}$ as collateral) will not change the equilibrium allocation. In the case of zero default costs, any bond with an intermediate collateral requirement can be replicated by holding a portfolio of the existing bonds, using the same amount of collateral. With positive default costs, any such bond is strictly dominated by a portfolio of existing bonds. Therefore with the set $\mathcal{J}_{\bar{a}}$ of bonds in place, the market is going to determine the collateral requirement, just as it determines the interest rate.

Since for each 'unregulated' tree a the collateral requirements of the S bonds in $\mathcal{J}_a$ is endogenous, it is useful to formally define our notion of a financial markets equilibrium with endogenous collateral requirements. We assume that for a set of trees $\widetilde{\mathcal{A}} \subset \mathcal{A}$ collateral requirements are endogenous, i.e. for each $a \in \widetilde{\mathcal{A}}$, there are at least S bonds for which this tree can be used as collateral. Our definition of equilibrium with endogenous collateral is then as follows.

DEFINITION 2 *Given an economy with initial tree holdings $(\theta^h(\sigma_0^*))_{h\in\mathcal{H}}$ and initial shock s_0 a financial markets equilibrium with endogenous collateral requirements for all trees $a \in \widetilde{\mathcal{A}} \subset \mathcal{A}$*

is a collection

$$((\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma))_{h\in\mathcal{H}}, \bar{q}_1(\sigma), \ldots, \bar{q}_A(\sigma), \bar{p}_1(\sigma), \ldots, \bar{p}_J(\sigma), (k_a^j(s^t))_{j\in\mathcal{J}, a\in a(j)})_{\sigma\in\Sigma}$$

satisfying the following conditions:

(1) Markets clear:

$$\sum_{h\in\mathcal{H}} \bar{\theta}^h(\sigma) = 1 \text{ and } \sum_{h\in\mathcal{H}} \bar{\phi}^h(\sigma) = 0 \text{ for all } \sigma \in \Sigma.$$

(2) For each agent h :

$$(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma)) \in \arg \max_{\theta\geq 0, \phi, c\geq 0} U_h(c) \text{ s.t. for all } s^t \in \Sigma$$

$$c(s^t) = e^h(s^t) + \sum_{j\in\mathcal{J}} \left([\phi_j(s^{t-1})]^+ r(s^t) + [\phi_j(s^{t-1})]^- f_j(s^t)\right) + \theta^h(s^{t-1}) \cdot (\bar{q}(s^t) + d(s^t)) -$$

$$\theta^h(s^t) \cdot \bar{q}(s^t)) - \phi^h(s^t) \cdot \bar{p}(s^t)$$

$$\theta_a^h(s^t) + \sum_{j\in\mathcal{J}} k_a^j(s^t)[\phi_j^h(\sigma)]^- \geq 0, a = 1, \ldots, A$$

(3) For all s^t, for each $\bar{a} \in \mathcal{A}$ and each direct successor node (s^t, s') there is a $j \in \mathcal{J}$ such that

$$k_{\bar{a}}^j(s^t) \left(q_a(s^t, s') + d(s^t, s')\right) = 1.$$

It will be useful to refer to the different bonds by their default probability. The way things are set up, the set $\mathcal{J}_{\bar{a}}$ contains the risk-free bond, as well as a bond that defaults in precisely one state, a bond that defaults in precisely two states etc. We refer to these bonds as the non-default (or risk-free bond) the 1-default bond, the 2-default bond etc. Without costs of default, some of these bonds will be traded in equilibrium but we will argue below that in our calibration moderate costs generally suffice to shut down trade in these bonds.

To fix ideas we sometimes assume that there is only one bond available for trade and that collateral requirements are the lowest possible that ensure no default in the next period (this is similar to the specification of collateral requirements in Kiyotaki and Moore (1997)). In this case, we have for the single bond $j = 1$ that

$$\min_{s^{t+1}\succ s^t} k_{a(j)}^j(s^t) \left(q_a(s^{t+1}) + d(s^{t+1})\right) = 1.$$

We refer to this bond as the 'risk-free' or 'default-free' bond. Formally, we would need to define financial markets equilibrium with a single default-free bond, but the definition is obvious and therefore omitted.

Finally we consider the case of regulated collateral. We assume that a regulator sets so-called 'haircuts'. These haircuts are the value of the collateral asset minus the value of the loan, divided by the value of the asset, i.e. the haircut $\bar{h}(s^t)$ satisfies

$$h_a^j(s^t) = \frac{k_a^j q_a(s^t) - p_j(s^t)}{k_a^j q_a(s^t)}.$$

If the haircut is regulated to be $\bar{h}_j^a(s)$ in shock s, we have that the collateral requirement at each node s^t is set to

$$k(s^t) = \frac{p_j(s^t)}{q_a(s^t)(1 - \bar{h}_a^j(s_t))}.$$

Alternatively to haircuts we could have considered margin-requirements which are simply defined as $\frac{k_a^j q_a(s^t) - p_j(s^t)}{p_j(s^t)}$. The advantage of using haircuts lies in the fact that they are bounded above by 1 while margins can become unbounded.

2.2 Calibration

This section discusses the calibration of the model's exogenous parameters.

2.2.1 Growth rates

We consider a growth economy with stochastic growth rates. The aggregate endowment at date-event s^t grows at the stochastic rate $g(s_{t+1})$ which (if no default cost are incurred) only depends on the new shock $s_{t+1} \in \mathcal{S}$, that is, if either $\lambda = 0$ or $f_j(s_{t+1}) = 1$ for all $j \in \mathcal{J}$, then

$$\frac{\bar{e}(s^{t+1})}{\bar{e}(s^t)} = g(s_{t+1})$$

for all date-events $s^t \in \Sigma$. There are $S = 6$ exogenous shocks. We declare the first three of them, $s = 1, 2, 3$, to be "disasters." We calibrate the disaster shocks to match the first three moments of the distribution of disasters in Barro and Jin (2009). Also following Barro and Jin, we choose transition probabilities such that the six exogenous shocks are i.i.d. The non-disaster shocks, $s = 4, 5, 6$, are then calibrated such that the entire distribution of growth rates matches both the average growth rate and business cycle fluctuations with a standard deviation of 2 percent of the U.S. economy. We sometimes find it convenient to call shock $s = 4$ a "recession" since $g(4) = 0.966$ indicates a moderate decrease in aggregate endowments. Table 1 provides the resulting growth rates and probability distribution for the six exogenous shocks of the economy.

Shock s	1	2	3	4	5	6
$g(s)$	0.566	0.717	0.867	0.966	1.025	1.089
$\pi(s)$	0.005	0.005	0.024	0.065	0.836	0.065

Table 1: Growth rates and distribution of exogenous shocks

In our results sections below we report that collateral requirements have quantitatively strong effects on equilibrium prices. Obviously, the question arises what portion of these effects is due to the large magnitude of the disaster shocks. We address this issue in the discussion of our results. In addition, Section 5 examines the equilibrium effects of collateral requirements for an economy with less severe disaster shocks.

2.2.2 Endowments and dividends

There are $H = 2$ types of agents in the economy, the first type, $h = 1$, being less risk-averse than the second. Each agent h receives a fixed share of aggregate endowments as individual endowments, that is, $e^h(s^t) = \eta^h \bar{e}(s^t)$. We assume that $\eta^1 = 0.092$, $\eta^2 = 0.828$. Agent 1 receives 10 percent of all individual endowments, and agent 2 receives the remaining 90 percent of all individual endowments. The remaining part of aggregate endowments enters the economy as dividends of Lucas trees, that is, $d_a(s^t) = \delta_a(s_t)\bar{e}(s^t)$ and $\sum_a \delta_a(s) = 0.08$ for all $s \in \mathcal{S}$.

Several comments on the distribution of the aggregate endowment are in order. First, we abstract from idiosyncratic income shocks because it is difficult to disentangle idiosyncratic and aggregate shocks for a model with two types of agents. We conjecture that our effects would likely be larger if we considered a model with a continuum of agents receiving i.i.d. idiosyncratic shocks as, for example, in Krusell and Smith (1997). Second, a dividend share of 8 percent may appear a little too low if one interprets the tree as consisting of both the aggregate stock market as well as housing wealth. However, this number is in line with the results in Chien and Lustig (2009). We conduct some sensitivity analysis below and, in particular, report many results also for the case $\sum_a \delta_a(s) = 0.15$ and thus $\eta^1 = 0.085$, $\eta^2 = 0.765$. Third, for simplicity we do not model trees' and other assets' dividends to have different stochastic characteristics as aggregate consumption. Section 5 discusses other specifications. Fourth, in Section 4 we examine an economy with two Lucas trees. For such economies, we want to interpret the first tree as aggregate housing and its dividends as housing services while we interpret the second tree as the aggregate stock market. Following Cecchetti et al. (1993), we calibrate dividends to be 4 percent of aggregate consumption which leaves housing services to be of the same size. In order to focus on the effects of collateral and margin requirements, we assume that the two trees have the exact same dividend payments, that is, in the absence of collateral constraints these two trees would be identical assets. Therefore, this calibration allows for a careful examination of the impact of different collateral properties of the two trees.

2.2.3 Utility parameters

The choice of an appropriate value for the IES is rather difficult. On the one hand, several studies that rely on micro-data find values of about $0.2 - 0.8$, see, for example, Attanasio and Weber (199x). On the other hand, Vissing-Jorgensen and Attanasio (2003) use data on stock owners only and conclude that the IES for such investors is likely to be above one. Barro and Jin (2009) find that for a successful calibration of a representative-agent asset-pricing model the IES needs to be larger than one.

In our benchmark calibration both agents have identical IES of 1.5, that is, $\rho^1 = \rho^2 = 1/3$. In our sensitivity analysis we also consider the case of both agents having an IES of 0.5. For this specification the quantitative results slightly change compared to the benchmark calibration, but the qualitative insights remain intact.

Agent 1 has a risk aversion of 0.5 and so $\alpha^1 = 0.5$ while agent 2's risk aversion is 6 and thus $\alpha^2 = -5$. Recall the weights for the two agents in the benchmark calibration, $\eta^1 = 0.092$ and $\eta^2 = 0.828$. The majority of the population is therefore very risk-averse, while 10 percent of households have low risk aversion. This heterogeneity of the risk aversion among the agents is the main driving force for volatility in the model. (Agent 1 wants to hold the risky assets in the economy and leverages to do so. In a disaster shock, his deleveraging leads to excess volatility.) In the equilibria of our model, the risky assets are mostly held by agent 1, but there are extended periods of time where also agent 2 holds part of the asset. Loosely speaking, we therefore choose the fraction of very risk-averse agents to match observed stock-market participation.

Finally, we set $\beta^h = 0.95$ for both $h = 1, 2$, which turns out to give us a good match for the annual risk-free rate.

3 Economies with a single Lucas tree

We first consider economies with a single Lucas tree available as collateral. We show that scarce collateral has a large effect on the volatility of this tree and examine how the magnitude of this effect depends on the specification of margin requirements. This section sets the stage for our analysis of economies with two trees in Section 4.

3.1 Collateral and volatility with a single risk-free bond

The starting point of our analysis is an economy with a single Lucas tree and a single bond. We impose the type 2 collateral requirement mentioned above, that is, the collateral requirements ensure that there is no default in equilibrium and the bond is risk-free. We calibrate this baseline model according to the calibration presented above.

For an evaluation of the quantitative effects of scarce collateral, we benchmark our results against those for two much simpler models. The model *B1: No bonds* is an economy with a single tree and no bond. Thus, agents in this economy cannot borrow. The model *B2: Unconstrained* is an economy in which agents can use their entire endowment as collateral. This model is equivalent to a model with natural borrowing constraints.

As a robustness check for the results in our baseline model we consider two additional models. The model *R1: Low IES* is identical to the baseline model with the exception that the IES is 0.5 instead of 1.5. Similarly, the model *R2: High dividend* is identical to the baseline model with the exception that aggregate dividends are 15 percent instead of 8 percent, that is, 15 percent of the aggregate wealth in the economy is collateralizable.

Table 2 reports four statistics for each of the five economies. (See Appendix A for a description of the estimation procedure.) Throughout the paper we measure tree-price volatility by the average standard deviation of tree returns over a long horizon. Another interesting measure is the average one-period-ahead conditional price volatility. These two measures are closely correlated for our models. In Table 2 we report both measures but omit the second one in the remainder of the paper. We also report average interest rates and equity premia. While our paper does not focus on an analysis of these measures, we do check them because we want to ensure that our calibration delivers reasonable values for these measures.

Model	Std returns	1-period price vol.	Risk-free rate	Equity premium
B1: No bonds	5.33	4.98	n/a	n/a
B2: Unconstrained	5.38	5.05	5.88	0.55
Baseline	8.14	7.54	1.10	3.86
R1: Low IES	7.20	6.72	1.75	4.18
R2: High dividend	7.68	7.18	1.77	3.48

Table 2: Five Economies with a single tree (all figures in percent)

The perhaps most striking result reported in Table 2 is that volatility in our baseline economy is about 50 percent larger than in the two benchmark models without borrowing (*B1: No bonds*) and with natural borrowing constraints (*B2: Unconstrained*), respectively. For example, the standard deviation of returns is 8.14 percent in the baseline economy but only 5.33 percent and 5.38 percent for the benchmark models *B1* and *B2*, respectively.[1] We can give a nice intuitive explanation for this finding. For this purpose we first interpret the results for the benchmark models *B1* and *B2* and then explain the mechanism that yields excess volatility in our baseline model.

Recall that in our calibration agents of type 1 are much less risk averse than type 2 agents. Therefore, in the long run, agent 1 holds the entire Lucas tree in model *B1* with no borrowing and agent 2 effectively lives in autarchy. As a result the tree price is determined entirely by the Euler equation of agent 1, and so the price volatility is as low as in the model with a representative agent. A similar intuition applies to the second benchmark model *B2*. In the long run, the less risk-averse agent 1 holds the entire tree during the vast majority of time periods. The much more risk-averse agent 2 only very rarely buys a small portion

[1]The stock return volatility in our baseline economy is considerably smaller than the volatility in U.S. data. For comparison, Lettau and Uhlig (2002) report that the quarterly standard deviation of returns of S&P-500 stocks in post-war US data is about 7.5 percent. Similarly, Fei et al. (2008) report an annual volatility of about 14.8 percent for the period January 1987 to May 2008. However, it is important to note that we want to interpret the aggregate tree as a mix of stocks and housing assets. The volatility of housing prices is U.S. data is much lower. Fei et al. (2008) report an annual volatility of the Case/Shiller housing price index of less than 3 percent (for January 1987 to May 2008). A similar comment applies to the equity premium. While the average risk-free rate roughly matches U.S. data, the equity premium is substantially lower than in the data. We discuss this point in more detail in Section 4 for an economy with two trees.

of the tree. And so the return volatility is of similar magnitude as in the model without borrowing. Also observe both the high risk-free rate and very low equity premium in the model *B2* with natural borrowing limits. Due to the loose borrowing limits the supply of the risk-free bond is rather large, agent 1 can borrow large amount against her holding of the entire tree. The large supply suppresses the bond price and thus leads to large bond returns.

Collateral constraints drastically increase the volatility in the standard incomplete markets model. Although the mechanism through which this excess volatility is generated has been described in the economics literature before (see, for example, Kiyotaki and Moore, 1997, or Ayiagari and Gertler, 1999), we find it helpful to illustrate this mechanism in the context of our baseline model. Figure 1 shows the typical behavior of five variables in the long run during a simulation for a time window of 200 periods. The first graph displays agent 1's holding of the Lucas tree. The second graph shows the normalized tree price, that is, the equilibrium price of the tree divided by aggregate consumption in the economy. The third graph shows the collateral premium (to be explained below). The last two graphs show the price and agent 1's holding of the risk-free bond, respectively.

In the model *B1* without borrowing, the less risk-averse agent 1 holds the tree at all times, her Euler equation prices the tree and her consumption moves exactly with the aggregate shocks. The normalized tree price remains constant. On the contrary, this price shows substantial swings in the presence of collateral constraints. In the sample displayed in Figure 1, the disaster shock $s = 3$ (smallest disaster with a drop of aggregate consumption of 13.3 percent) occurs in periods 71 and 155 while disaster shock 2 occurs in period 168 and disaster shock 1 (worst disaster) hits the economy in period 50. In periods with these bad shocks the tree price drops substantially.

When a disaster shock occurs, both the current dividend and the expected net present value of all future dividends of the tree decrease. As a result the price of the tree drops, but in the absence of further effects, the normalized price should remain the same. (That's exactly what happens in the benchmark model *B1*.) In our baseline economy with collateral constraints, however, additional effects occur in equilibrium. For example, if the worst shock $s = 1$ hits the economy, then the entire financial wealth of the less risk-averse agent 1 is wiped out if the collateral constraint was binding in the previous period. She is left with her individual endowment. The financial wealth of the more risk-averse agent 2, however, is unaffected. In fact, his financial wealth relative to his individual endowment is comparatively large. This wealth distribution strongly affects the equilibrium. First, agent 1 being restricted by the collateral constraint can no longer own the entire tree but must sell some of her tree holdings to agent 2. Agent 2 is much more risk-averse than agent 1 and dislikes the risky tree. Therefore, in equilibrium, the tree price must drop substantially for agent 2 to buy a substantial portion of the tree from agent 1. This additional drop in the price of the tree reinforces the very causal chain just described, i.e. as agent 1 gets poorer, he has to sell a bigger share of the tree, which in turn depresses its price further. Secondly,

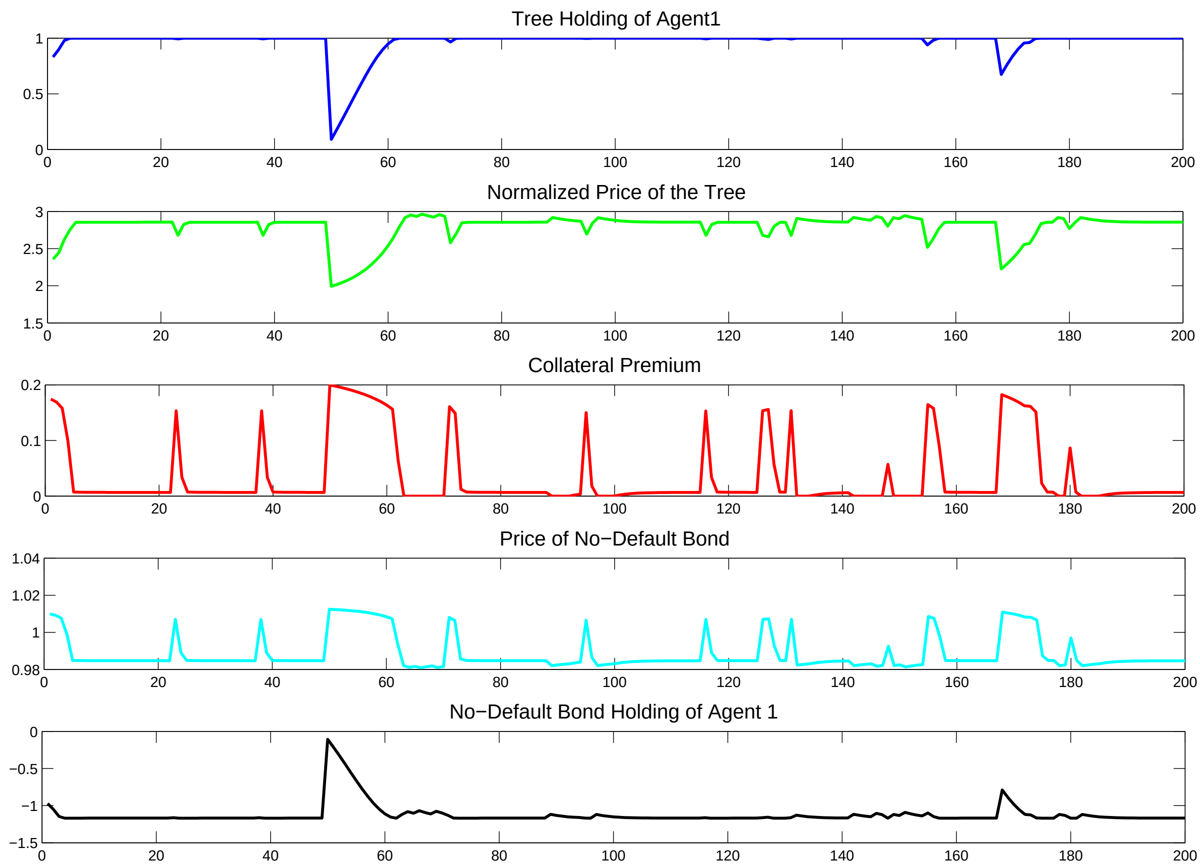

Figure 1: Snapshot from a simulation of the baseline model

agent 1 finances her remaining tree investment partially by a reduction in consumption and partially by borrowing a moderate amount from agent 2. Agent 2, in contrast, would like to invest much of his remaining financial wealth in the risk-free bond. However, agent 1 now owns substantially less collateral than before and so cannot borrow as much as before. The volume of the bond trading decreases substantially and the price of the bond increases so much that the interest rate becomes negative. In the absence of this general equilibrium effect, which results in cheap credit for agent 1, the price of the tree would decrease even more.

Figure 1 shows that the described effects are huge for shock $s = 1$ but they are also very large for shock 2. Note that the prices are normalized prices, so the drop of the actual tree price is much larger than displayed in the figure. In disaster shock 1, agent 1 is forced to sell almost the entire tree and the normalized price drops by almost 30 percent (the actual price drops by approximately 60 percent). In shock 2 she sells less than half of the tree but the price effect is still substantial. In shock 3 the effect is still clearly visible, although the agent only sells off very little of her tree.

The figure also shows that the multiplier effect of collateral constraints on asset prices does not only occur in the disaster state. All remaining price drops in Figure 1 can be identified with recessions, shock $s = 4$. The price of the tree already drops substantially below its 'fundamental value', agent 1 sells only a tiny amounts of the tree (barely visible in the figure), but the expectation of a huge price drop in the disaster state depresses the price today.

Finally, it is important to note that when agent 1 is constrained but would not need to sell the stock, the price of the stock would actually increase because of the collateral premium. In Figure 1 we also show the one period ahead collateral premium on the stock, i.e. the increase in the stock price in the current period that would result if agent 1 was unconstrained, given stock prices in the future. The figure shows that each time when the price of the tree decreases because the collateral constraint becomes binding, there is actually a quantitatively significant counter-acting effect. Interestingly, the magnitude of this effect does not change much with the intensity of the shocks and with the actual drop in the price of the stock. For example, around period 25 a recession hits the economy. The sell-off in the tree is barely visible but as explained above the tree price decreases. The collateral premium shoots up to 0.15. This observation will play an important role below when we consider the case where there are two trees but only one of them can be used as collateral.

Comparing the case of $\delta = 0.08$ to $\delta = 0.15$ reveals that volatility is substantially somewhat higher if there is less collateral but the basic effect remains intact even if the fraction of collateralizable wealth is 15 percent . It is clear that as δ becomes large, volatility must eventually decrease: In the extreme this is obvious: The unconstrained case in our calibration is nothing else but a $\delta = 1$. What is surprising is that the difference is somewhat small in the region between $\delta = 0.08$ and $\delta = 0.15$. In order to investigate this point further it is useful to plot stock price volatility as well as probability of the collateral constraint being binding as a function of δ.

Figure 2 illustrates this point – in the figure, we vary δ and depict the tree's average return volatility as well as the fraction of times the collateral constraint is binding for agent 1 (i.e. the probability of constraint being binding). For very small values of δ there is not much collateral in the economy and the constraint is almost always binding. On the other hand, the stock is so small that agent 1 does not have to start selling the stock, even if the economy is hit by an extremely bad aggregate shock. Return volatility is relatively small. As δ increases the probability of the collateral constraint being binding decreases rapidly but the effects of it being binding becomes larger. There is an interior maximum for the stock-return volatility at around $\delta = 0.07$. Although the constraint is much less often binding than for a smaller tree, the trade-off between agent 1 being forced to sell the tree and agent 1 getting into this situation leads to a maximum volatility. After that, the constraint becomes binding much less frequently and eventually at $\delta = 1$ stock return volatility is very

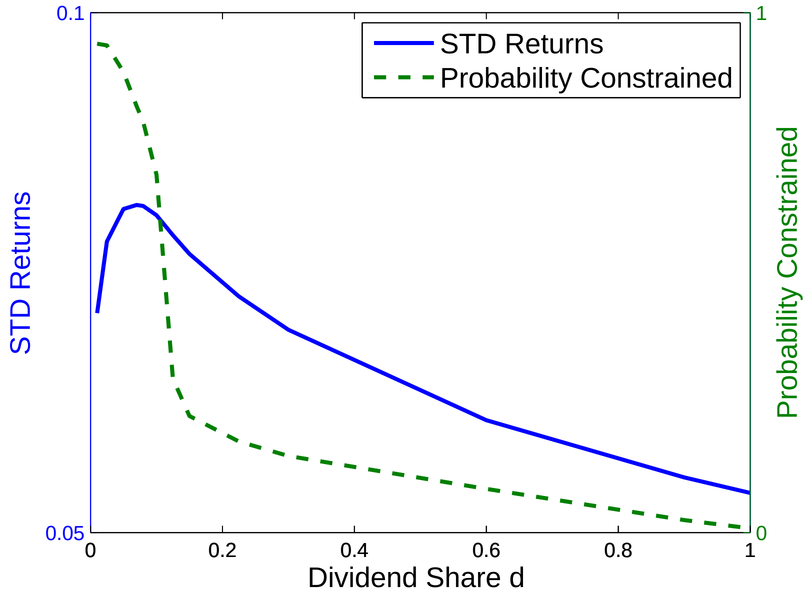

Figure 2: Volatility as a function of the dividend share

low, simply because the collateral constraint never binds the multiplier effect of collateral plays no role. This is identical to the case of natural borrowing constraints and a binding collateral constraint would imply zero consumption for the borrower.

In our baseline calibration we set both agents intertemporal elasticity of substitution equal to 1.5. For comparison, we also report results here for an IES of 0.5. The volatility is substantially lower than in the baseline case, but still clearly much higher than in an economy without collateral constraints (while not reported in the table, the standard deviation of stock returns there is 5.34). In the following we will concentrate on a IES of 1.5 – as mentioned above, the empirical evidence is mixed and our results do not depend crucially on this parameter. For a lower IES, effects are similar, but quantitatively less important. For low IES, there is an additional unwanted effect that as one agent holds most of the wealth (i.e. the other becomes poor) asset prices increase because of the desire of the rich agent to save. This effect is absent when the IES is set to 1.5 which we will do for the remainder of our analysis.

3.2 Endogenous margins

If margin requirements are endogenous, given our calibration, there are in principle 5 bonds available for trade (as explained above, these are characterized by the shocks in which they are on the 'verge of default' and we denote them by no-default bond, 1-default bond, 2-default bond etc.). It turns out that in 'normal times', i.e. outside of disaster-states practically only the no-default bond, i.e. the one with the largest margin requirement, is traded (there is tiny trade in the 1-default bond in recessions, but this is quantitatively negligible). Figure 3 shows the portfolio holdings and the normalized tree price along the same simulated series of shocks as in Figure 1 above.

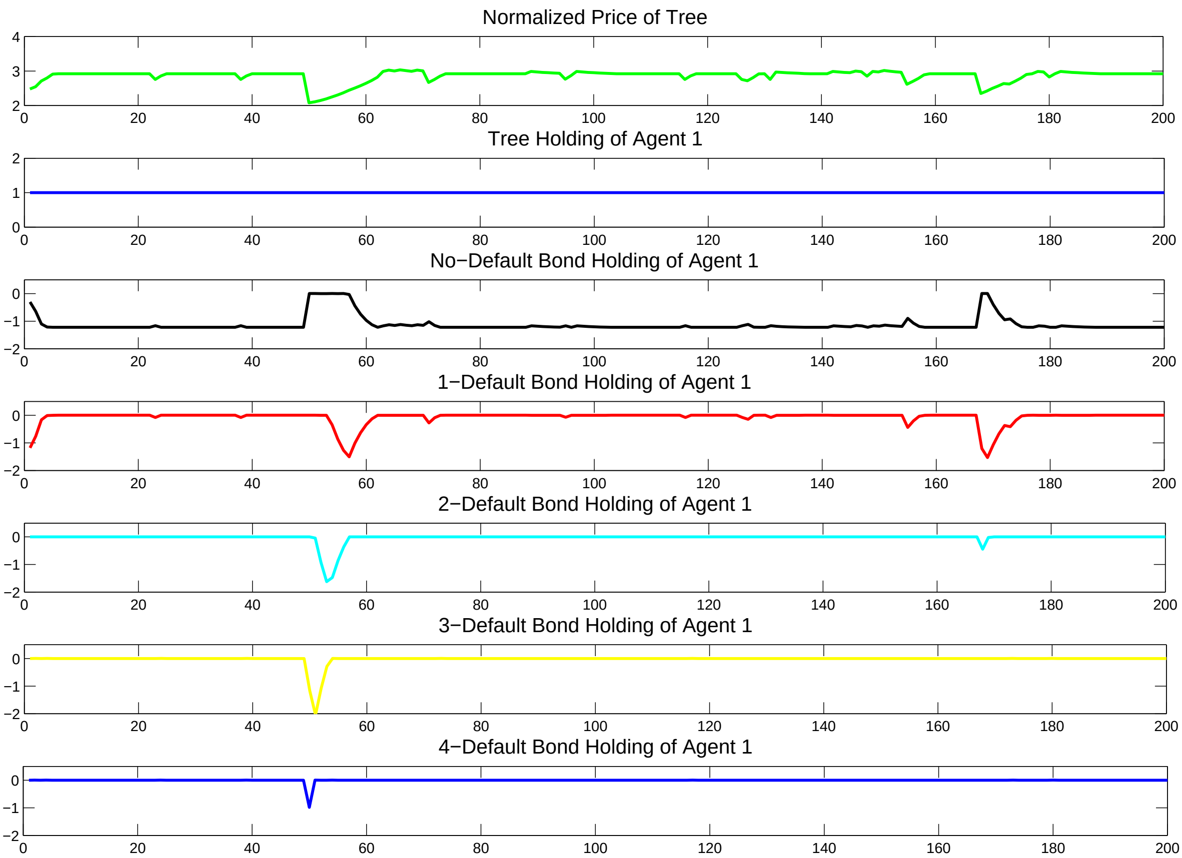

Figure 3: Snapshot from a simulation of the model with 1 tree and 5 bonds

The figure shows that disaster states are the main reason for trade in defaultable bonds. In 'normal' times (i.e. outside of disaster states and sufficiently long after the last disaster shock), the risk-averse agent can buy the risk-free bond and in exchange give the risky stock to agent 1. Given the way we set up the model, this is to be expected: The risk-averse agent is seeking to buy an asset that insures him against bad aggregate shocks – only the risk-free bond can play this role. However, the other bonds play an important role once times go bad. If a disaster state hits, agent 1 now does not have to sell the stock but can raise additional funds and shift some of the tree's risk to the other agent by selling off defaultable bonds. It is true that agent 2 will demand a high interest rate for them – but as long as they do not default in all states, they are still less risky than the tree and therefore preferred by the risk-averse agent. In fact, in our calibration, without default costs, the less risk-averse agent, agent 1, holds the tree at all times. Even in the worst disaster shock 1, in the figure it occurs in period 50, agent 1 is able to hold on to the entire tree and just sell the 5-default and the 4-default bond to agent 2. As the economy recovers, agent 1 sells the 1-default bond to agent 2 and holds a short-position in this bond for approximately 10 years until his wealth has recovered sufficiently so that he is able to leverage exclusively in the default free bond.

Unfortunately, it seems somewhat counterfactual that in bad times agents start trading bonds that are likely to default. There are several explanations for why the effect in our model cannot often be observed in data. One is clearly that bad times are often persistent (and not iid as in our calibration). Perhaps more importantly, default is typically costly. We show in the next subsection that relatively small costs of default shut down trade in defaultable bonds.

Despite the fact that fire sales in the tree no longer occur, negative aggregate shocks still have large effects on asset prices. Figure 3 shows that not only in disaster shock 1 the price of the asset collapses but, similarly to Figure1, in all negative shocks. The reason for this is that by selling off the defaultable bonds to the risk-averse agent 2, agent1 shifts the tree's risk to agent 2, and the tree must be priced accordingly. This becomes obvious if one would consider a case where pay-outs of the tree in shock 5 and 6 are identical: In this case, the tree and the 5-default bond have identical pay-outs and hence it should be irrelevant for the price of the tree who holds it, i.e. whether agent 1 holds it financed by a short position in the 5-default bond or agent 2 holds it directly. In our case, pay-outs in shocks 5 and 6 are sufficiently close so that the differences in price-effects appear small.

Nevertheless, as the following table shows, the presence of endogenous margins does reduce tree-price volatility substantially. In Table 3 we decompose the effect and show how much each bond adds.

	One bond	Two bonds	Three bonds	Four bonds	All bonds
Std returns	8.14	7.87	7.84	7.84	7.84

Table 3: The effect of endogenous margins

The table shows that in particular, the presence of a bond that defaults in shock 1 (when the economy shrinks by 43.4 percent) decreases volatility significantly. A third bond still has significant effects, while all other bonds seem to be almost irrelevant. It is clear that the possibility of default reduces volatility. Endogenous margin requirements allow agent 1 to trade away larger parts of the tail risk involved in his tree to agent 2 if he is constrained. He has to sell less (or in as in our case none) of the tree and the drop in price is significantly less severe. However, it is interesting to note that only bonds that default in state 1 or in state 2, but pay back in full in all other states have significant effects on volatility. It is clear that a bond that defaults in all three disaster state is not a very attractive asset for trade in this economy. The return that the risk averse agent 2 will demand to hold this bond will not be significantly different than the return he demands to hold the tree. Therefore, the trade in this bond is small and the effect on price volatility negligible.

3.2.1 Costly default

As discussed in the introduction, our treatment of default is somewhat unsatisfactory since it neglects both private and social costs of default. We model the costs of default as explained in Section 2 above. While this is clearly an oversimplified way to approach the issue, it gives us some idea what is needed in a model so that bad shocks cannot be alleviated by default bonds. In Table 4 we describe how trading volume in the default bonds decreases as a function of λ. Trading volume is computed as the average absolute bond holding (of agent 1, which is the same as of agent 2) over the simulation path.

	$\lambda = 0$	$\lambda = 0.01$	$\lambda = 0.05$	$\lambda = 0.10$	$\lambda = 0.2$	$\lambda = 0.25$
Std Tree	7.84	7.87	7.98	8.12	8.15	8.14
Total trading	1.260	1.236	1.183	1.161	1.126	1.123
No-default bond	1.110	1.099	1.076	1.076	1.099	1.123
1-default bond	0.084	0.080	0.075	0.085	0.027	0
2-default bond	0.034	0.034	0.032	0	0	0
3-default bond	0.026	0.023	0	0	0	0
4-default bond	0.006	0	0	0	0	0

Table 4: The effect of default costs

As we have seen above, for the case of no default costs (i.e. $\lambda = 0$), there is substantial trading volume in default-bonds. By continuity, the trading volume remains high if costs of default are very small. However, the table also shows that for default costs of 25 percent, trade in the default bonds disappears completely. Recall that the cost is proportional to the difference of the face value of the bond and the value of the underlying collateral. A proportional cost of 25 percent here therefore means a much smaller cost as a fraction of the underlying collateral. Campbell et al. (2010) find an average 'foreclosure discount' of 27 percent - this is measured as a percentage of the total value of the house, not the difference between house value and face-value of the debt which is often much lower. A value for λ of 0.25 therefore seems certainly realistic (if anything too small if one has in mind the housing market) – for this case only the default free bond is traded and the equilibrium prices and allocations are identical to the base-line case above.

For realistic values of default costs, default does not occur in our equilibrium and agents endogenously trade only the risk-free bond. This is clearly somewhat counterfactual, as default is a fact of life and certainly in the recent crisis default turned out to be an aggregate phenomenon. It is clear that the introduction of idiosyncratic shocks might lead to default on the individual level, but this then would have no aggregate consequences. One explanation is obviously that the cost of default is much less than 25 percent. At a default cost of 10 percent only two bonds are traded: The 1-default bond and the no default bond. Table 4 also shows that volatility increases as costs of default are introduced. This is clear since with high enough costs, we get back to the base-line case of agents only trading the non-default bond. However, it is interesting to note that with default costs of 20 percent and some trade in the 1-default bond, return volatility of the tree is actually slightly higher than in our baseline case where only the no-default bond is traded. This is due to the fact that default implies real losses in our setup and that therefore with very high default costs trade in the 1-default bond makes the worst disaster shock even worse since the default leads to a further drop in output.

Finally, the table reveals that trading volume in the 1-default bond actually remains stable up to default costs of around 10 percent. It only drops sharply, after all other bond markets are shut down. This is intuitively clear: This bond insures agent 1 against the worst disaster state. This shock happens with extremely low probability, but when it happens the consequences for this stock-holder are severe. A default cost of 5 percent still means that the 1-default bond is a very attractive asset in this economy. This is also consistent with the bond holdings in Figure 3 which we observe without default costs. The trade in the 3-5 default bond only occurs in very few periods and is shut down quickly. Under default costs it will then be replaced with trade in the 1-default bond.

3.3 Volatility with regulated haircuts

Finally we consider the case of exogenous margin requirements (or in our case, haircuts). We assume that there is a regulatory agency that sets minimal haircuts (as is the case for stocks). We first assume that haircuts are set to be constant across all shocks. As margin requirements become tighter, i.e. haircuts become larger, we observe two effects: Clearly the amount of leverage decreases leading to less de-leveraging in the disaster shocks and to smaller price effects. On the other hand, the collateral constraint is more likely to become binding, yielding in principle a higher volatility of the stock. Initially volatility increases as haircuts become tighter, but at a haircut of about 70 percent, the volatility reaches its maximum. A further tightening of margins then decreases volatility substantially. It is clear from the case of no borrowing that eventually, if haircuts are so high that they preclude all borrowing, volatility will become extremely low. What is more interesting is that starting at the risk-free bond a regulation of haircuts initially increases volatility slightly - for the risk-free bond, haircuts obviously vary across shocks and time, but if haircuts are 60 percent they turn out to be uniformly larger than for the no-default bond. In our calibration, this leads to an increase of volatility.

On the other hand, we consider the thought experiment that margins are only regulated in booms while in recessions and crises, they are left to the market. This leads to a substantial decrease of volatility compared to the case of only the risk-free bond (or the case of endogenous margins and costs of default). In particular, we assume that in shocks

1 through 4 collateral requirements are at the level of the risk-free bond, while a regulator is setting haircuts in the shocks with positive growth. For simplicity we assume that they are set to the same level in shocks 5 and 6.

Figure 4 illustrates the differences between shock-dependent regulation and deterministic regulation of haircuts.

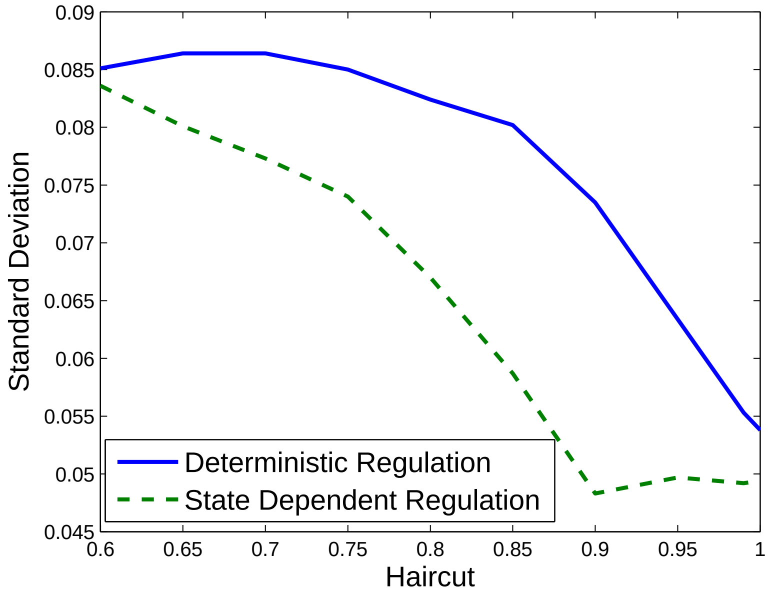

Figure 4: Volatility as a function of the haircut

The figure illustrates our earlier point that a deterministic regulation of haircuts initially leads to a slight increase of tree-return volatility. In fact, volatility is more or less flat until a haircut of 75 percent. Only beyond that can one achieve a significant decrease of volatility with state independent regulation. On the other hand, the table also shows that by regulating haircuts only in booms, the volatility of the tree price can be reduced substantially. In particular, for haircuts of around 80 percent volatility already falls to 6.5 percent (while being above 8 percent in the case of unregulated margins or state-independent regulation). Also note, that for haircuts in booms of about 90 percent volatility reaches its global minimum. At these haircuts there is still some borrowing even in the boom states, but agent 1 can typically no longer afford to buy the entire tree. In the case of bad shocks, he then actually purchases some of the tree which stabilizes the price. One could expect that with such high exogenous haircuts, the borrower, agent 1, is forced to de-lever in shocks 5 and 6. Since this actually happens in the boom states, it leads to a decrease in tree return volatility.

4 Two trees

Our analysis so far focused on the case of a single tree, representing aggregate collateralizable wealth in the economy. In reality, households trade in various assets and durable goods. Some of them, like houses can be used as collateral very easily, others like stocks can only be used as collateral for loans with high margin-requirements and typically very high interest rates (see Willen and Kubler (2006)), and still others, like works of art, cannot be used as collateral at all. In this section, we examine a model with two trees. For simplicity, we assume that the two trees have identical cash-flows and distinguish themselves only by the extent to which they can be used as collateral. This will allow for a clean analysis of the effect of collateral. We consider two different cases. First, we assume that tree 1 can be used as collateral with endogenous margin requirements, while tree 2 cannot be used as collateral at all. We then also allow the second tree to serve as collateral, but we assume that the collateral requirements on this loan are exogenously regulated. In both cases we find that the two assets' price dynamics a completely different, despite the fact that they have identical cash-flows. Furthermore, we show that tightening the margin requirements on regulated trees has substantial impact on the return volatility of the non-regulated trees. This turns out to be a quantitatively important effect which needs to be considered in any policy-discussion on the regulation of margin-requirements.

4.1 Only one tree can be used as collateral

We first consider the case where the second tree cannot be used as collateral at all. Somewhat surprisingly that dramatically increases the volatility of returns on this tree (compared to the case where it can be used as collateral) as well as its equity premium.

It is useful to consider the analogue of Figure 3 for this case. Figure 5 shows tree prices and portfolio-holdings along 'our' sample path. Three effects can be observed. First, price volatility for tree 1 is much lower. Secondly, price volatility for tree 2 is larger than before. Lastly, there is no trade in bonds that default in more than 1 states and the 1-default bond is only traded in the worst disaster shock 1. As before, agent 1 holds tree 1 the entire time, but now he does not have to resort to defaultable bonds in bad times but he simply sells off the second tree.

In Table 5 we report moments of the trees' returns.

In the first half of the table we report moments that arise if only the risk-free bond is available for trade on tree 1, the second half of the table reports moments for the case of no default costs, when margin requirements are endogenous. As before, relatively small costs of default shut down trade in all default bonds and we are back to the first case. In fact, as Figure 3.1 (??) shows the effect of default costs on average returns and volatility are very small. In this calibration costs of $\lambda = ...$ (J+M???) suffice to shut down all trade in default bonds. In any case, since the numbers are sufficiently similar we focus our discussion on the case with only the default-free bond.

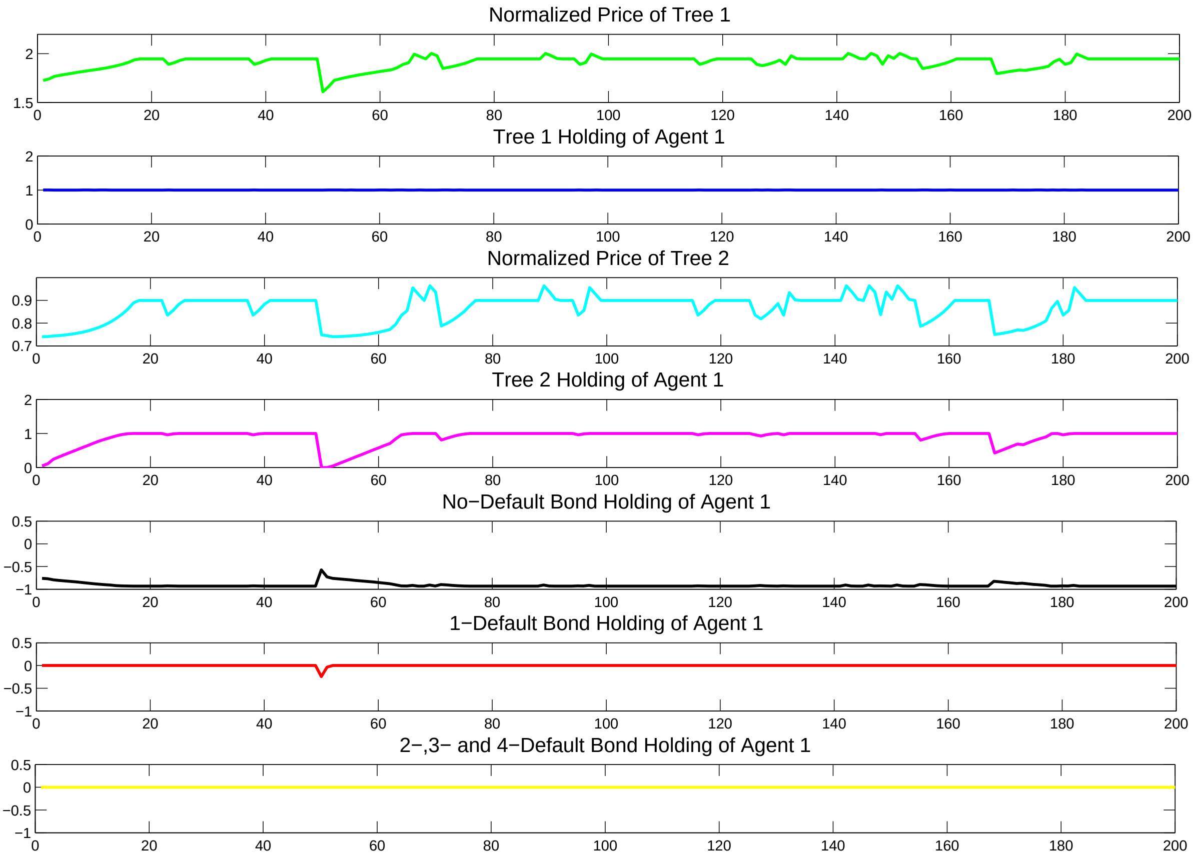

Figure 5: Snapshot from a simulation of the model with 2 trees and 5 bonds

The table shows that if only one half of the tree can be collateralized, the interest rate is much lower and the (average) equity premium much higher than in the case of a single tree. Moreover, the excess return of tree 2 - the tree that cannot be used as collateral - is now similar to what can be observed in the data. Tree 1 is more valuable to agents because of its collateral value. Given that both trees have identical cash-flows, an agent can only be induced to hold tree 2 if it pays a higher average return. Therefore its excess return must be larger. It is of course a quantitative issue by how much it has to be larger and it turns out that in our calibration where the market price of risk is realistic, the effect is very large – the average excess return of the second tree is now comparable to that observed in US stock market data.

Note that compared to the baseline case of a single tree the standard deviation of returns of both trees is lower. However, tree 2's standard deviation is similar to the tree volatility in the baseline case, while tree 1 which is being used as collateral now has significantly lower volatility. The explanation for this is simple. When faced with financial difficulties, agent 1 first sells tree 2. He holds on to tree 1 as long as he can, because this tree allows him to

	Std returns	Std returns agg	Risk-free rate	Equity-premium	EP agg
Tree 1, 1 bond	6.64	7.04	0.38	3.69	4.50
Tree 2	8.05			6.31	
Tree 1, all bonds	6.56	6.99	0.37	3.63	4.46
Tree 2	7.98			6.31	

Table 5: Moments of trees' returns (tree 1 collateralizable, tree 2 not)

hold a short-position in the bonds. When being sold, the price effects for tree 1 and tree 2 are similar however: As the risk-averse agent 2 is forced to hold a tree, the price collapses – this is true for both trees. An important difference between the two trees is, however, that once agent 1 is constrained, the price of tree 1 tends to rise because of the collateral premium. As in the case of a single tree, this is quantitatively significant and counteracts the potential drop in the price. Therefore, the return volatility of the first tree decreases significantly compared to our baseline case.

While we do not want to push the interpretation of our results too far, it is worthwhile to note that a natural interpretation of the two trees is the aggregate stock market versus the aggregate housing market. As Willen and Kubler (2006) report, it is often very difficult to use stocks as collateral. It is clear that volatility in the stock-market is much higher than in the housing market. This interpretation clearly should be taken with some caution, since we do not really have a good model of the housing market – this would include transaction costs, non-divisibilities etc. But it is interesting to point out that the equity premium for tree 2 is similar to what can be observed in the data for stock-returns and it is clear that volatility of housing returns is much smaller than that of stock returns.

4.2 One tree is regulated

Clearly our above assumption that tree 2 cannot be used as collateral at all is unrealistic. Stocks can be used as collateral, however, haircuts are regulated and large, and interest rates are much higher than mortgage rates. We therefore assume now that haircuts for tree 2 are set exogenously while collateral requirements for tree 1 are endogenous. Throughout this section, we assume default costs of $\lambda =$??? which suffice to shut down all trade in defaultable bonds. As in Section 4.1. the results for the case without default costs are very similar and discussing this case does not add much to this section.

Clearly, if haircuts for tree 2 are set very high, we are back to the above case where this tree cannot be used as collateral at all. On the other hand, if both trees are unregulated we are back in our baseline case from Section 3 - both trees have identical volatility and average returns. In Figure 6 we now plot the volatility of both trees' returns as a function of the haircut set for tree 2.

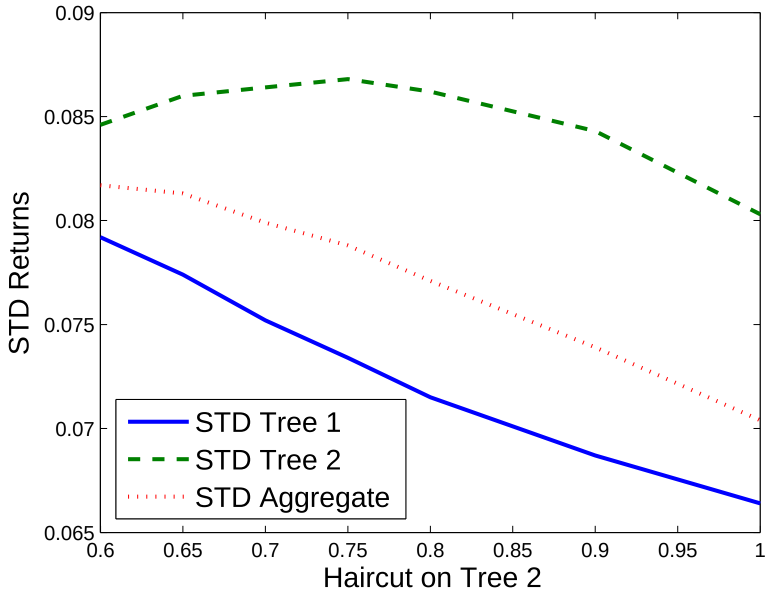

Figure 6: Volatility of tree 1 and 2 as a function of the haircut on tree 2

Figure 6 reveals a surprising effect. In the figure, we start off with a haircut of tree 2 set to 60 percent – in most states this is clearly above the unregulated haircut, so the volatility of tree 2 is already higher than that of tree 1. If haircuts on tree 2 are now increased, the volatility of this tree's return initially increases, while the volatility of the freely collateralizable tree substantially decreases. The volatility of tree 2 is largest when it can be used as collateral but when exogenous haircuts are quite high (about 75 percent). After this, also the volatility of tree 2 decreases until we are at haircuts of 100 percent which means that tree 2 cannot be used as collateral at all and we are back to the case above.

The quantitatively most interesting case is clearly a regulated haircut of 75 percent. At this point, the volatility of tree 2 is above 8.6 percent while the volatility of tree 2 is below 7.5 percent. Overall volatility is still high, but the regulation of tree 2 has substantial effects on its own volatility as well as on the volatility of the other, unregulated tree.

The interpretation is clear. As tree 2 becomes more unattractive as collateral, i.e. as haircuts on this tree increase, agent 1 sells tree 1 less and less often and tree 2 more and more often. Tree 1's return volatility decreases since fire-sales occur less often. As we saw in Section 3 above, if the total size of the tree decreases, its volatility decreases since agent 1 can hold on to the tree even in bad shocks – this is what happens here. The opposite is true for tree 2. Naively, one might conclude that tree 2's volatility increases monotonically in its haircut. But obviously one also has to consider the effect that we already observed in Section 3: As the total amount of collateral in the economy goes down, there is less leverage and fewer fire sales. This is then true both for tree 1 and for tree 2 – as the haircuts for tree 2 become sufficiently high, this tree is sold less and less often by agent 1.

Moreover, as the haircut on tree 2 becomes large, price-effects of fire-sales on this tree become smaller: With low haircuts, tree 2 is not only more attractive to agent 1 because of its risky payout, it is also more attractive because of its collateral-premium. Since agent 2 does not use any tree as collateral, a sale of the tree two agent 2 causes large price-movements.

Related to this point, it is also interesting to consider the excess returns of the two trees as a function of haircut on tree 2. Figure 7 shows that in this case the relation is monotone for tree 2

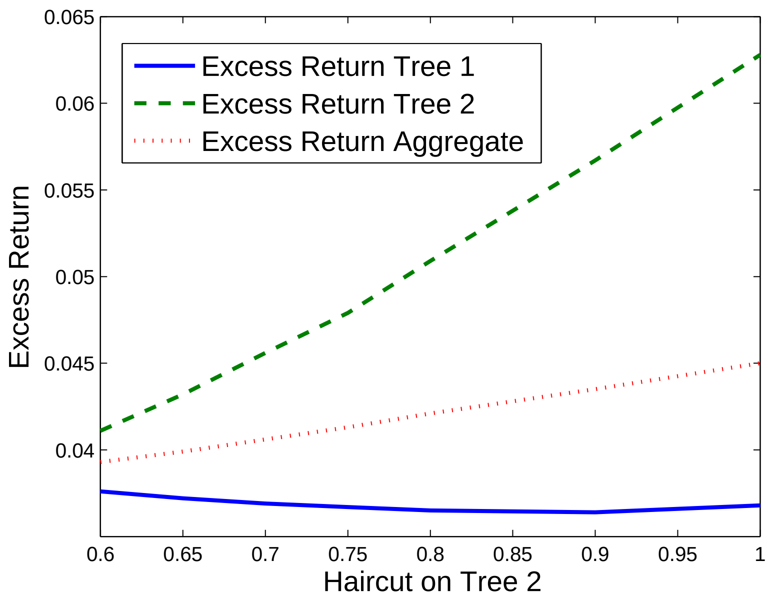

Figure 7: Excess returns

As its haircut increases, the collateral premium and the price of the tree decrease and the average return therefore goes up. For tree 1 average excess returns remain more or less constant. They initially decrease slightly then increase slightly. Aggregate excess returns increase, but clearly the quantitatively striking effect is on the returns of tree 2. Collateral constraints and regulated haircuts clearly have a quantitatively huge effects on asset prices in this economy.

4.2.1 Optimal regulation of tree 2

The above analysis shows that it is difficult to keep volatility low for both assets, if regulation is state-independent and targets only one tree. With moderate haircuts, a change in haircuts always reduces volatility of one asset at the expense of the other. We now analyze whether a state-dependent regulation of the second tree can solve this dilemma. Figure 8 shows that the volatilities of both assets are monotonically decreasing in the haircut imposed on the regulated tree in good times. Hence, increasing haircuts now reduces the volatilities of both assets, and it does reduce aggregate asset market volatility much more. For instance, it drops by 10%, if state-dependent haircuts are increased from 0.6 to 0.7, while such an increase would bring about a reduction of only 2% in case of state-independent regulation. Thus, concerning regulation, the upshot from the single tree economy is strongly confirmed by the two tree analysis: regulation is much more efficient, if it is state-dependent.

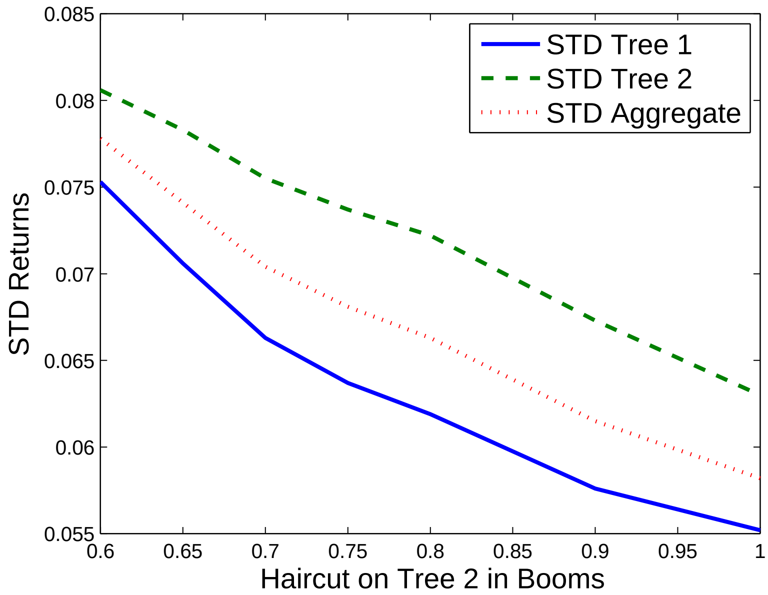

Figure 8: Volatility as a function of the haircut on tree 2 in booms

5 Sensitivity analysis and extensions

As in any quantitative study our results above hinge crucially on the parameterization of the economy. In particular, our modeling decision to include the possibility of disaster states plays an important role for our results. In this section, we report ...

5.1 Preferences and endowments

No hope without disaster state. Even very high risk aversion does not cut it...

Appendix

A Details on Computations

A.1 Time iteration algorithm

The algorithm used to solve all versions of the model is based on Brumm and Grill (2010). Equilibrium policy functions are computed by iterating on the per-period equilibrium conditions, which are transformed into a system of equations. We use KNITRO to solve this system of equations for each grid point. Policy functions are approximated by piecewise linear functions. By using fractions of financial wealth as the endogenous state variables, the dimension of the state space is equal to the number of agents minus one. Hence with two agents, the model has an endogenous state space of one dimension only. This makes computations much easier than in Brumm and Grill (2010), where two and three dimensional problems are solved. In particular, in one dimension reasonable accuracy may be achieved without adapting the grid to the kinks. For the reported results we used 320 or 640 grid points depending on the complexity of the version of the model, which results in average (relative) Euler errors with order of magnitude 10^{-4}, while maximal errors are about ten times higher. If the number of gridpoints is increased to a few thousands, then Euler errors fall about one order of magnitude. However, the considered moments only change by about 0.1 percent. Hence, using 320 or 640 points provides a solution which is precise enough for our purposes. Compared to other models the ratio of Euler errors to the number of grid points used might seem large. However, note that due to the number of assets and inequality constraints our model is numerically much harder to handle than standard models. For example, in the version with one tree and five bonds, eleven assets are needed (as long and short positions in bonds have to the treated as separate assets) and we have to impose eleven inequality constraints per agent.

A.2 Simulations

The moments reported in the paper are averages of 50 different simulations with a length of 10.000 periods each (of which the first 100 are dropped). This is enough to let the law of large numbers do its job even for the rare disasters.

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