Taxes, Debts, and Redistribution

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Motivation

- How costly are high levels of government debt?
- Should the gov't try to reduce its initial high debt? If so, how quickly?
- How should tax rates, transfers, and government debt respond to aggregate shocks?

Motivation

Analysis with complete markets is well known:

- Smooth distortionary costs of raising revenue
- Labor taxes are (approximately) constant
- Arrow securities used to finance all expenditure needs

Our focus: Markets less than fully complete and there are limits to redistribution

Key ingredients

- **Heterogeneity:** Agents are heterogeneous in productivities and assets
- **Instruments:** A tax system that is linear in labor income and an intercept that is uniform across agents
- **Markets:** All agents trade a *single* security whose payoff might depend on aggregate shocks

Characterize optimal tax rate, transfers and asset purchases

Findings: Theory

- Welfare depends on distribution of asset positions across agents.
  - Gross levels of debt do not matter
- Ergodic distribution and speed of convergence of debts and taxes depend on:
  - **Spanning ability:** More correlated returns imply faster convergence and smaller spread in debts and taxes.
  - **Redistribution concerns:** Higher welfare weights on poor agents imply lower asset accumulation.

Analytical results for quasilinear preferences and some extensions to more general preferences

Findings: Quantitative

Exercise:

- Calibrate to match U.S. inequality in assets and earnings
- Match two business cycle features:
  - Earning drops are larger for poor agents in recessions
  - Realized returns on government assets are uncorrelated with output

Findings:

- Cyclical properties of optimal policies consistent with existing U.S. policies
- **Key difference:** Under optimal allocation debt is repaid much slower

Related literature

- Representative agent incomplete market economies
  - Barro (1974, 1979), Aiyagari et al (2002), Faraglia-Marcet-Scott (2012), Farhi (2010), etc
- Representative agent complete market economies
  - Lucas-Stokey (1983), Chari-Kehoe (1999), etc
- Heterogeneous agents with complete markets
  - Werning (2007), Azzimonti-Francisco-Krusell (2008)

Environment

- **Uncertainty**: Markov aggregate shocks s_t
- **Demography**: N types of infinitely lived agents (mass n_i)
- **Technology**: Output $\sum_i n_i \theta_{i,t} l_{i,t}$ is linear in labor supplies.
- **Preferences** (Households)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t} \right)$$

- **Preferences** (Planner): Given Pareto weights $\{\omega_i\}$

$$\mathbb{E}_0 \sum_i \omega_i \sum_{t=0}^{\infty} \beta^t U_t^i \left(c_{i,t}, l_{i,t} \right)$$

- **Asset markets**: A risky bond with payoffs $P_t = \mathbb{P}(s_t | s_{t-1})$

Environment, II

- **Affine Taxes**: Agent i's tax bill

$$-T_t + \tau_t \theta_{i,t} l_{i,t}$$

- **Budget constraints** Let $R_{t-1,t} = \frac{P_t}{q_{t-1}}$
  - Agent i: $c_{i,t} + b_{i,t} = (1 - \tau_t)\,\theta_{i,t} l_{i,t} + R_{t-1,t} b_{i,t-1} + T_t$
  - Government: $g_t + B_t + T_t = \tau_t \sum_i n_i \theta_{i,t} l_{i,t} + R_{t-1,t} B_{t-1}$
- **Market Clearing**
  - Goods: $\sum_i n_i c_{i,t} + g_t = \sum_i n_i \theta_{i,t} l_{i,t}$
  - Assets: $\sum_i n_i b_{i,t} + B_t = 0$
- **Initial conditions**: $(\{b_{i,-1}, B_{-1}\}_i, s_{-1})$

Ramsey Problem

Definition

Allocation, price system, government policy: Standard

Definition

Competitive equilibrium: Given $(\{b_{i,-1}\}_i, B_{-1}, s_{-1})$ and $\{\tau_t, T_t\}_{t=0}^{\infty}$, a competitive equilibrium is an allocation and price system such that households are optimizing and markets clear

Definition

Optimal competitive equilibrium: A welfare-maximizing competitive equilibrium for a given $(\{b_{i,-1}\}_i, B_{-1}, s_{-1})$

Irrelevance of initial debt

Define *net assets* : $\tilde{b}_{i,t} = b_{i,t} - b_{1,t}$

Proposition

For any pair of initial distributions $\left(\{b'_{i,-1}\}_i, B'_{-1}\right)$ *and* $\left(\{b''_{i,-1}\}_i, B''_{-1}\right)$

$$b'_{i,-1} - b'_{1,-1} = b''_i - b''_{1,-1}$$

the welfare at the optimal allocations are the same.

Normalize assets of least productive agent to zero.

Optimal policy: Instruments and active channels

- Respond to aggregate shocks using:
  1. Fluctuations in asset returns
  2. Taxes, transfers

- Main considerations:
  1. Degree of market completeness
  2. Concerns for redistribution

Use quasi-linear setting to derive analytical results

Quasilinear setting

- Assume:
  1. Quasi Linear preferences :$u(c, l) = c - \frac{l^{1+\gamma}}{1+\gamma}$
  2. 2 types of agents with $\theta_1 > \theta_2 = 0$
  3. IID aggregate shocks to expenditures
  4. $c_{2,t} \geq 0$

- Disentangles spanning and redistribution concerns:

Lemma

Let (ω, n) be the Pareto weight and mass of the productive agents. If $\omega > n\left(\frac{1+\gamma}{\gamma}\right)$ then $T_t = 0 \quad \forall t \geq 0$.

Low redistribution concerns, use only fluctuations in assets returns to hedge the aggregate risk.

Degree of market incompleteness

Decompose the set of payoff vectors: $\mathcal{P} \equiv \mathcal{P}^* \cup \mathcal{P}^{*c}$ where

$$\mathcal{P}^* = \left\{ P^*(s) : P^*(s) = 1 + \frac{\beta}{B^*}(g(s) - \mathbb{E}g) \text{ for some } B^* \in [\overline{B}, \underline{B}] \right\}$$

Payoffs $P^* \in \mathcal{P}^*$ perfectly hedge fluctuations in net-of interest deficits at debt level B^* :

$$B^*(P) = \beta \frac{\mathrm{var}(g(s))}{\mathrm{cov}(P^*(s), g(s))}.$$

Invariant distribution: for $\omega > n\left(\frac{1+\gamma}{\gamma}\right)$

Proposition

The behavior of government assets under a Ramsey plan is characterized as follows:

1. *If $P \in \mathcal{P}^*$ then government assets converge to a degenerate steady state*

$$\lim_t B_t = B^*(P) \quad a.s \quad \forall B_{-1}.$$

   There is $\tau^*(P)$ such that $\lim_t \tau_t = \tau^*(P) \quad a.s \quad \forall B_{-1}.$

2. *If $P(s) \notin \mathcal{P}^*$ there exists an invariant distribution of government assets with the property,*

$$\forall \epsilon > 0, \quad \Pr\{B_t < \underline{B} + \epsilon \;\; and \; B_t > \overline{B} - \epsilon \quad i.o\} = 1.$$

   There is a $\tau(B)$ such that the tax rate $\tau_t = \hat{\tau}(B_t)$ and $\hat{\tau}' < 0$.

Approximation to ergodic distribution

- For $P(s) \in \mathcal{P}^*$, we can replicate complete markets perfectly asymptotically with assets $B^*(P)$
- Use this to construct an approximation for the ergodic distribution of debt and taxes of an economy with $P(s)$ "close" enough to $\mathcal{P}^*$.
- In particular split $P(s)$

$$P(s) = \hat{P}(s) + P^*(s)$$

where $P^*(s) \in \mathcal{P}^*$ and $\hat{P}(s)$ is orthogonal to $g(s)$.

Linearize policy rules around $\hat{P} = \mathbf{0}$ and study the properties of the ergodic distribution generated by such rules.

More

Ergodic distribution

Proposition

For an economy with payoffs $P(s)$, the linearized policy rules induce a ergodic distribution of government debt with

- **Mean:** *The ergodic mean of asset distribution, $\mathbb{E}(B)$, satisfies*

$$\mathbb{E}(B) = B^*(P^*)$$

- **Variance:** *The ergodic coefficient of variation of government assets B is*

$$\frac{\sigma(B)}{\mathbb{E}(B)} \leq \sqrt{\frac{\mathrm{var}(\hat{P}(s))}{\mathrm{var}(P^*(s))}}$$

- **Convergence rate:** *The rate of convergence to of the mean to its ergodic value is described by,*

$$\mathbb{E}_{t-1}(B_t - B^*) = \frac{1}{1 + \mathrm{var}(P)\mathrm{corr}^2(P, g)}(B_{t-1} - B^*)$$

Ergodic Distribution

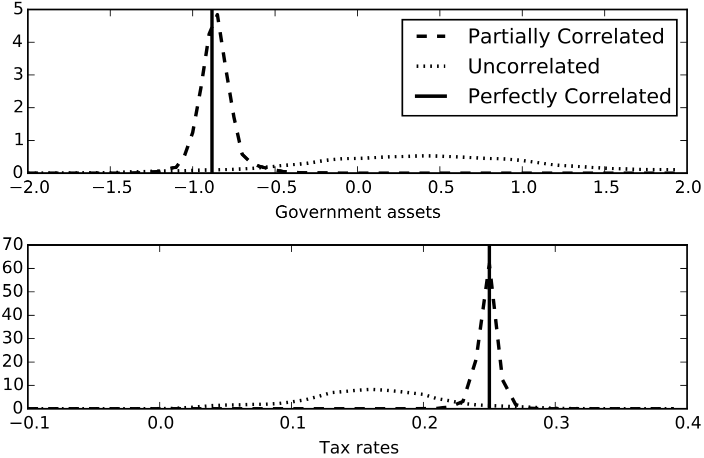


Figure: Ergodic distribution for government assets B_t and the labor tax rate τ_t in the representative agent quasilinear economy for three different asset payoff vectors P.

Redistribution concerns: for $\omega < n\left(\frac{1+\gamma}{\gamma}\right)$

Proposition

If $P \notin \mathcal{P}^*$ and $\min_s\{P(s)\} > \beta$, there exist a $\mathcal{B}(\omega)$ satisfying $\mathcal{B}'(\omega) > 0$ and we have two cases:

1. *If $B_{-1} > \mathcal{B}(\omega)$ then*

$$T_t > 0, \quad \tau_t = \tau^*(\omega), \text{ and } B_t = B_{-1} \quad \forall t \geq 0.$$

2. *If $B_{-1} \leq \mathcal{B}(\omega)$ then*

$$T_t > 0 \text{ i.o.}, \quad \lim_t \tau_t = \tau^*(\omega) \text{ and } \lim_t B_t = \mathcal{B}(\omega) \quad \text{a.s.}$$

More cases when $P \in \mathcal{P}^*$ since we have two absorbing points.

Remarks

- Balancing costs of fluctuations in tax rates and transfer
  - fluctuations in taxes is costly: deadweight loss
  - fluctuations in transfers is costly: deviations from target level of redistribution
- For large ω transfers are costly as the planner gives resources to unproductive agents
- For low ω, transfers are used:
  - For low initial debt, interior solution: All shocks hedged by transfers
  - For high debt, accumulate assets until costs of transfers are equalized to costs of collecting labor taxes
- The more redistributory the planner is:
  - bigger average tax rates and transfers
  - less need to accumulate assets for precautionary reasons

Risk aversion

- Endogenous fluctuations in returns as prices depend on marginal utility
- However, same general flavor as quasi-linear economy
  - cost of fluctuations in transfers comes from cost of fluctuation in $U_c \iff$ similar to multiplier on constraint $c \geq 0$ in quasi-linear case
  - Similar spanning considerations if we redefine $\tilde{P}(s) = \frac{P(s)U_c(s)}{\mathbb{E}U_c(s)P(s)}$

$$\frac{b^i_{t-1}}{\beta} P_t = c^i_t - T_t - (1-\tau_t)\theta_{i,t} l_{i,t} + b^i_t$$

More

Risk aversion

- Endogenous fluctuations in returns as prices depend on marginal utility
- However, same general flavor as quasi-linear economy
  - cost of fluctuations in transfers comes from cost of fluctuation in $U_c \iff$ similar to multiplier on constraint $c \geq 0$ in quasi-linear case
  - Similar spanning considerations if we redefine $\tilde{P}(s) = \frac{P(s)U_c(s)}{\mathbb{E}U_c(s)P(s)}$

$$\frac{x_{t-1}^i}{\beta}\frac{P_t U_{c,t}^i}{\mathbb{E}_{t-1}P_t U_{c,t}^i} = U_{c,t}^i(c_t^i - T_t) + U_{l,t}^i l_t^i + x_t^i$$

Where $x_t^i = U_{c,t}^i b_t^i$

More

Quantitative Exercise

Need to take a quantitative stand on

- Degree of market incompleteness: Payoff structure
- Dynamics of inequality: Skill shocks

Use U.S data on

- Cross sectional distribution of labor earnings, wealth: Levels and movements over business cycle
- Realized returns on government assets

Key challenge: Earnings, returns etc. are endogenous in the model

Quantitative Exercise: Strategy

1. Compute a competitive equilibrium with tax-debt policies fitted to observed US policies
2. Use endogenous outcomes in the competitive equilibrium to calibrate:
   - **changes in skill distribution:** match changes in distribution of earnings over the business cycle
   - **payoff shocks:** match how realized returns co move with output
3. Set Pareto weights such that optimal taxes are similar to those observed in data
4. Solve for the Ramsey allocation

Competitive equilibrium: Tax-Debt policy rules

- Policy rules for tax and debt:

$$\log D_t = (1 - \rho_{D,D}) \log \bar{D} + \rho_{D,D} \log D_{t-1} + \rho_{D,Y} \log Y_t,$$

$$\tau_t = (1 - \rho_{\tau,\tau})\bar{\tau} + \rho_{\tau,\tau}\tau_{t-1} + \rho_{\tau,Y} \log Y_t,$$

- Estimate using market value of U.S. federal debt, average marginal tax rates from TAXSIM.
- Transfers obtained as a residual from budget constraint

Parameter	Value	Parameter	Value
$\bar{D}$	0.6 (.006)	$\bar{\tau}$	0.24 (0.002)
$\rho_{D,D}$	0.411 (0.17)	$\rho_{\tau,\tau}$	0.20 (0.18)
$\rho_{D,Y}$	-0.72 (0.45)	$\rho_{\tau,Y}$	0.02 (0.09)

Table: OLS estimates for tax and debt policy rules. The numbers in brackets are standard errors.

Quantitative Exercise: Primitives

- Use 9 types of agents: Capture 10th-90th deciles of working U.S. population
- Initial asset inequality to match net asset distribution [SCF, 1978-2010].
- Shocks:
  - Aggregate productivity:

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \sigma_\theta \epsilon_{\theta,t},$$

  - Skill distribution:

$$\log \theta_{i,t} = \log \bar{\theta}_i + \varepsilon_t[1 + (.9 - \mathcal{Q}(i))m]$$

  - Asset payoffs:

$$p_t = 1 + \chi \epsilon_{\theta,t} + \sigma_p \epsilon_{p,t}$$

  $\epsilon_{p,t}, \epsilon_{\theta,t}$ are i.i.d standard normal
- Pareto weights set to match average optimal tax rate to 24%

$$\omega_i = d_0 + d_1 \bar{\theta}_i.$$

Solution Method

- In non-stochastic limit: distribution of individual states constant.
- Approximate policy rules locally using perturbation theory
  - Around current distribution of individual states
- Create an approximation to the global solution using a sequence of local approximations.
- Computation time grows linearly in the number of agents

More

Outcomes under optimal policy: Long run

Long run properties: Comparative statics

Implications from QL theory:

1. Speed of convergence is faster when payoffs are more volatile or correlated with fundamentals
2. The spread of ergodic distribution debt and taxes increases with the variance of the orthogonal component in payoffs.
3. More redistributive planners accumulate fewer assets

Long run properties: Speed of convergence

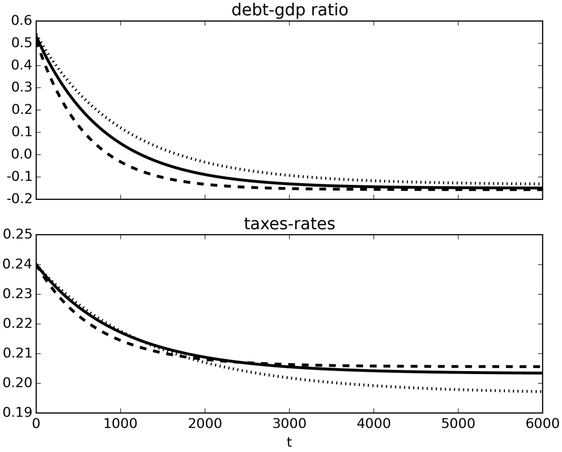


Figure: The dashed, bold and dotted lines plot conditional mean paths for debt to gdp and tax rates for a common sequence of shocks at different values of χ that generate $Corr[UcP, \epsilon] = \{-0.551, -0.394(\text{benchmark}), -0.242\}$ and $Corr[R, y] = \{-0.23, -0.051(\text{benchmark}), 0.13\}$respectively.

Speed of convergence: Back-of-the-envelope-calculation

How informative is the QL formula? Use $\tilde{P}(s) = \frac{U_c P(s)}{\mathbb{E} U_c(s) P(s)}$

$$\text{Half Life}(B) = \frac{\log(0.5)}{\log\left(\frac{1}{1+\text{var}[\tilde{P}]\text{corr}^2[\tilde{P},\epsilon_\theta]}\right)}$$

- Formula predicts a half life of about 2000 years
- Numerical simulations of quantitative model: Half life of debt is about
  - 1000 periods for the benchmark calibration $\sigma_P = 0.031$
  - 2000 periods if we shut down the orthogonal component $\sigma_P = 0$
- Role of second-order terms that are absent in the formula

Long run properties: Spread of debt, taxes

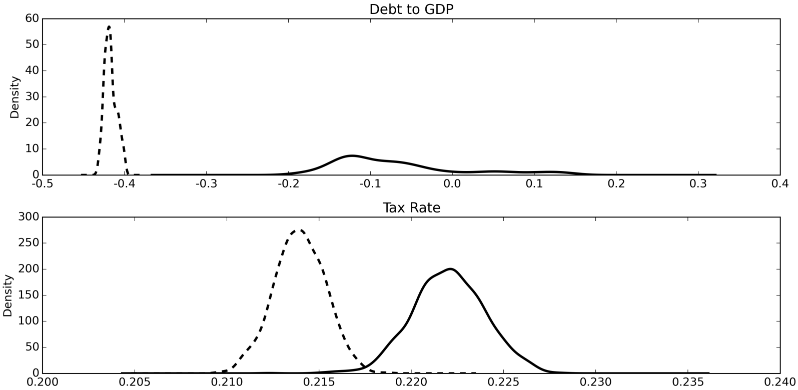


Figure: Ergodic distribution of the debt to gdp ratio and tax rates for $\sigma_P = 0$ (dashed line), and $\sigma_P = 0.031$ (bold line).

Long run properties: Redistribution and mean assets

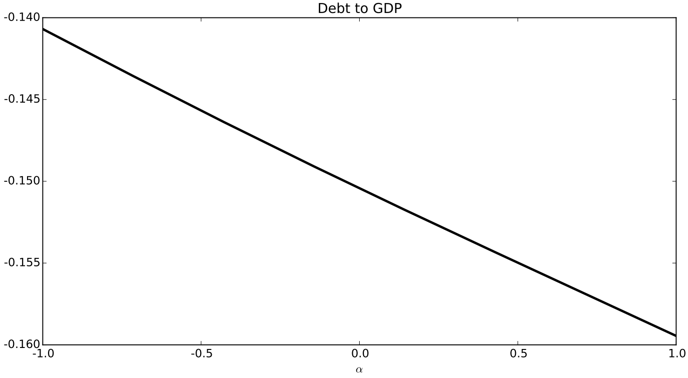


Figure: Long run debt as a function of α: Higher α represent higher weights on productive agents.

Outcomes under optimal policy: Short run

Impulse Responses - Taxes

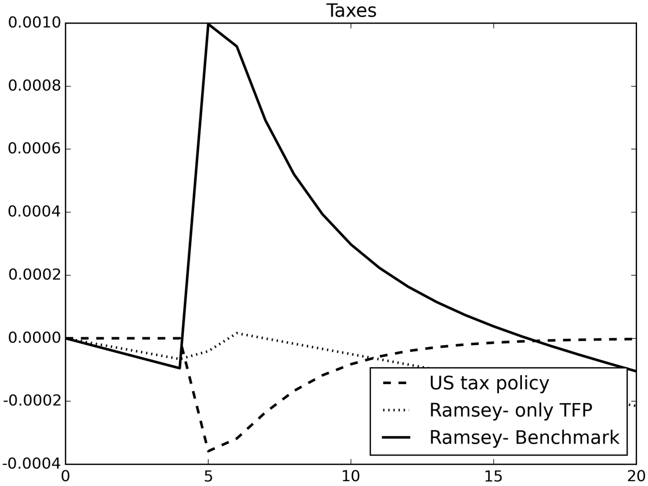


Figure: Impulse responses to tax rates to a negative one standard deviation aggregate productivity shock $\epsilon_{\theta,t}$

Impulse Responses - Debt

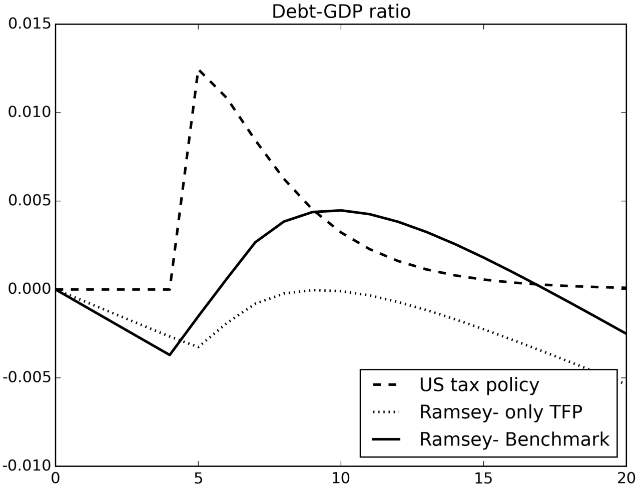


Figure: Impulse responses to debt-gdp ratio to a negative one standard deviation aggregate productivity shock $\epsilon_{\theta,t}$

Impulse Responses - Transfers

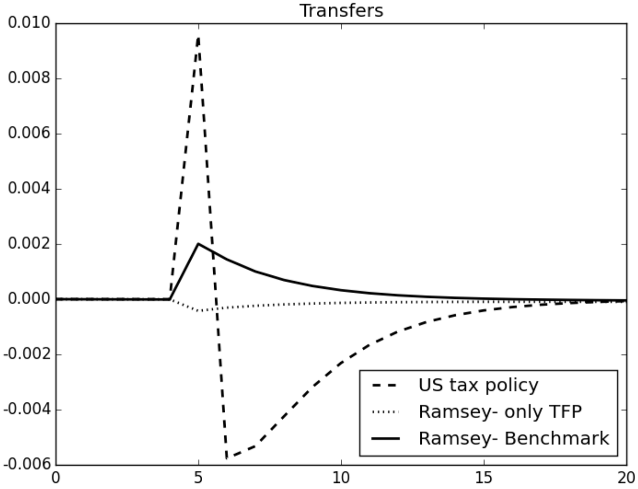


Figure: Impulse responses to transfers-gdp ratio for a negative one standard deviation aggregate productivity shock $\epsilon_{\theta,t}$

Takeaways

- Taxes are essentially smooth, both in data and optimal allocation
- Debt is repaid much slower under the optimal allocation
- Transfers as a residual adjust faster in data relative to the optimal allocation and overshoot
- In absence of inequality shocks, model implies correlation of transfers with output has opposite sign.

Conclusion

- Size of government debt alone is not informative $\implies$ need to know the net distribution of assets in the economy
- How payoffs correlate with fundamentals important in determining long run policies
- Ignoring heterogeneity produces misleading results about size and direction of the optimal policy response

Bellman Equation: Quasilinear

$$V(B_{-}) = \max_{\{c_1(s), c_2(s), l_1(s), B(s)\}_s} \sum_s \pi(s) \left\{ \omega \left(c_1(s) - \frac{l_1(s)^{1+\gamma}}{1+\gamma} \right) + (1-\omega) c_2(s) + \beta V(B(s)) \right\}$$

where the maximization is subject to

$$c_1(s) - c_2(s) - n^{-1} B(s) = l_1(s)^{1+\gamma} - n^{-1} \beta^{-1} P(s) B_{-},$$

$$n c_1(s) + (1-n) c_2(s) + g(s) = n \theta l_1(s),$$

$$c_2(s) \geq 0,$$

$$\underline{B} \leq B(s) \leq \overline{B}.$$

Bellman Equation: Risk Aversion

$$V(\boldsymbol{x}, \boldsymbol{\rho}, s_-) = \max_{\{a(s), x'(s), \rho'(s)\}} \sum_s \pi(s|s_-) \left(\left[\sum_i \omega_i U^i(s) \right] + \beta V(\boldsymbol{x}'(s), \boldsymbol{\rho}'(s), s) \right)$$

where the maximization is subject to

$$U_c^I(s)\left[c_i(s) - c_I(s)\right] + \left(\frac{U_l^i(s) l_i(s)}{U_c^i(s)} U_c^I(s) - l_I(s) U_l^I(s) \right) + x_i'(s) = \frac{x_i U_c^i(s) P(s)}{\beta \mathbb{E} U_c^i P} \text{ for all } s, i < I$$

$$\frac{\mathbb{E}_{s_-} P \boldsymbol{U}_c^i}{\mathbb{E}_{s_-} P \boldsymbol{U}_c^I} = \rho_i \text{ for all } i < I$$

$$\frac{U_l^i(s)}{\theta_i(s) U_c^i(s)} = \frac{U_l^I(s)}{\theta_I(s) U_c^I(s)} \text{ for all } s, i < I$$

$$\sum_i n_i c_i(s) + g(s) = \sum_i n_i \theta_i(s) l_i(s) \quad \forall s$$

$$\rho_i'(s) = \frac{U_c^i(s)}{U_c^I(s)} \text{ for all } s, i < I$$

Back

Ergodic distribution: Linear approximation

- For a given $P(s), g(s)$, we can compress the equilibrium conditions to two functions $b(\mu_{-})$ and a law of motion $\mu(s|\mu_{-})$
- Instead of approximating near a deterministic steady state we,
  - explicitly recognize that policy rules depend on payoffs: $\mu(s|\mu_{-}, \{P(s)\}_s)$ and $b(\mu_{-}, \{P(s)\}_s)$
  - take the first order expansion with respect to both μ_{-} and $\{P(s)\}$ around the vector $(\bar{\mu}, \{\bar{P}(s)\}_s)$ where $\bar{P}(s) \in \mathcal{P}^*$:
- The choice of $\bar{P}(s)$ is pinned down by

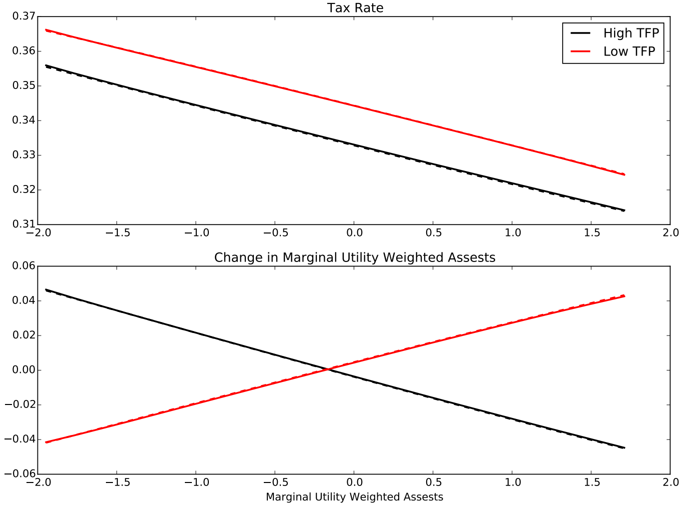
$$\min_{\tilde{P} \in \mathcal{P}^*} \sum_s \pi(s)(P(s) - \tilde{P}(s))^2.$$

- The law of motion approximated by

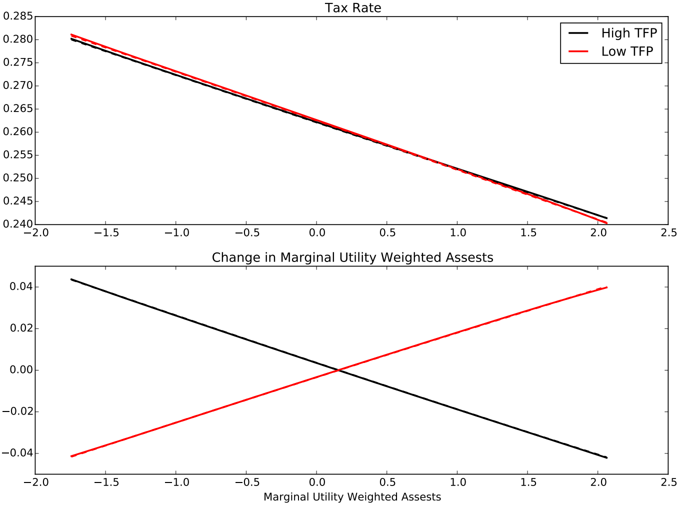
$$\mu_t - \mu^* = (\mu_{t-1} - \mu^*)B(s_t) + C(s_t)$$

back

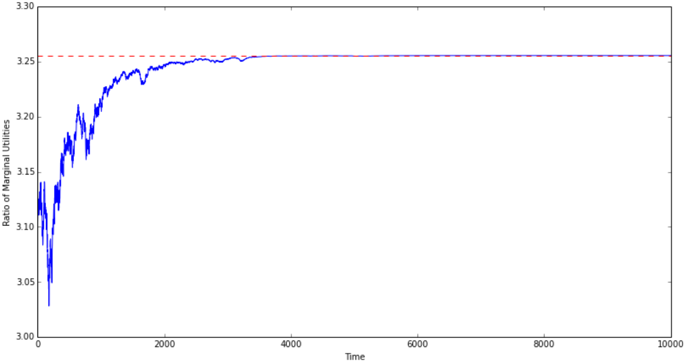
Accuracy: Short Run ($\rho = 3$)



Accuracy: Short Run ($\rho = 3.3$)



Accuracy: Long Run



Ramsey problem: Recursive formulation

Split into two parts

1. $\mathbf{t \geq 1}$: Ex-ante continuation problem with state variables $(\boldsymbol{x}, \boldsymbol{\rho}, s_{-})$

$$\boldsymbol{x} = \beta^{-1}\left(U^2_{c,t-1}\tilde{b}_{2,t-1}, ..., U^I_{c,t-1}\tilde{b}_{I,t-1}\right)$$

$$\boldsymbol{\rho} = \left(U^2_{c,t-1}/U^1_{c,t-1}, ..., U^I_{c,t-1}/U^1_{c,t-1}\right)$$

2. $\mathbf{t = 0}$: Ex-post initial problem with state variables $(\tilde{\boldsymbol{b}}_{-1}, s_0)$

Bellman Equation for $t \geq 1$

$$V(\boldsymbol{x}, \boldsymbol{\rho}, s_{-}) = \max_{c_i(s), l_i(s), x'(s), \rho'(s)} \sum_s \Pr(s|s_{-}) \left(\left[\sum_i \pi_i \alpha_i U^i(s) \right] + \beta V(\boldsymbol{x}'(s), \boldsymbol{\rho}'(s), s) \right)$$

where the maximization is subject to

$$U_c^i(s)\left[c_i(s) - c_1(s)\right] + U_c^i(s)\left(\frac{U_l^i(s)}{U_c^i(s)} l_i(s) - \frac{U_l^1(s)}{U_c^1(s)} l_1(s)\right) + \beta x_i'(s) = \frac{x_i P(s|s_{-}) U_c^i(s)}{\mathbb{E}_{s_{-}} \boldsymbol{U}_c^i P} \text{ for all } s, i$$

$$\frac{\mathbb{E}_{s_{-}} P \boldsymbol{U}_c^i}{\mathbb{E}_{s_{-}} P \boldsymbol{U}_c^1} = \rho_i \text{ for all } i \geq 2$$

$$\frac{U_l^i(s)}{\theta_i(s) U_c^i(s)} = \frac{U_l^1(s)}{\theta_1(s) U_c^1(s)} \text{ for all } s, i \geq 2$$

$$\sum_i n_i c_i(s) + g(s) = \sum_i n_i \theta_i(s) l_i(s) \quad \forall s$$

$$\rho_i'(s) = \frac{U_c^i(s)}{U_c^1(s)} \text{ for all } s, i \geq 2$$

$$\underline{x}_i(s; \boldsymbol{x}, \boldsymbol{\rho}, s_{-}) \leq x_i(s) \leq \bar{x}_i(s; \boldsymbol{x}, \boldsymbol{\rho}, s_{-})$$

Bellman equation for $t = 0$

$$V_0\left(\{\tilde{b}_{i,-1}\}_{i=2}^{I}, s_0\right) = \max_{c_{i,0}, l_{i,0}, x_0, \rho_0} \sum_i \pi_i \alpha_i U^i(c_{i,0}, l_{i,0}) + \beta V\left(x_0, \rho_0, s_0\right)$$

where the maximization is subject to

$$U^i_{c,0}\left[c_{i,0} - c_{1,0}\right] + U^i_{c,0}\left(\frac{U^i_{l,0}}{U^i_{c,0}} l_{i,0} - \frac{U^1_{l,0}}{U^1_{c,0}} l_{1,0}\right) + \beta x_{i,0} = U^i_{c,0}\tilde{b}_{i,-1}P(s_0) \text{ for all } i \geq 2$$

$$\frac{U^i_{l,0}}{\theta_{i,0}U^i_{c,0}} = \frac{U^1_{l,0}}{\theta_{1,0}U^{1,0}_{c}} \text{ for all } i \geq 2$$

$$\sum_i n_i c_{i,0} + g_0 = \sum_i n_i \theta_{i,0} l_{i,0}$$

$$\rho_{i,0} = \frac{U^i_{c,0}}{U^1_{c,0}} \text{ for all } i \geq 2$$