Hụi: A Case Study of A Sequential Double Auction of Capital

Timothy P. Hubbard,[1] Harry J. Paarsch,[2] and Will M. Wright[3]

[1]Colby College
[2]University of Melbourne
[3]National Australia Bank

Motivation and Introduction

- Among Vietnamese immigrants, in countries such as Australia and New Zealand, there exists an healthy distrust of formal institutions, including banks.
- Moreover, even when this distrust can be overcome, many of these immigrants simply do not have long enough credit histories to qualify for conventional small-business loans.
- Yet one of the principal ways in which immigrants accumulate capital is by starting and growing small businesses, such as laundries and restaurants as well as neighbourhood markets and repair shops.

Motivation and Introduction

- Using experience gathered by their ancestors over generations in their home countries, these immigrants often employ alternative institutions that allow them to borrow and to lend among themselves within their communities.
- One such institution is the hụi which, as we shall argue later, is essentially a sequential double auction.
- An hụi allows a group of immigrants to pool scarce financial resources, and then to allocate these resources among potentially lucrative investments.

In a Typical Hụi

- N people form a coöperative.
- Each participant deposits a minimum sum u with the *banker*.
- In each month thereafter, until each participant has had his turn to win, a first-price, sealed-bid auction is held to determine the implicit interest rate paid.
- We refer to each auction in the hụi as a *round* of the hụi.
- After the winner for a given round has been determined, only the winning bid is revealed.

In a Given Round t,

- Each participant must choose a bid s which is the discount below the deposited amount u he would be willing to accept from each remaining participant in that round.
- The participant submitting the highest bid w_t wins that round of the hụi, and is excluded from participating in all subsequent rounds.
- The winner receives the capital

$$[u + (t-1) \times u + (N-t) \times (u - w_t)].$$

- The winner must pay u to each winner in the remaining $(N-t)$ rounds.

An Example

- $N = 4$
- $u = \$300$

- Round 1 bids: $\{\$12, \$10, \$8, \$6\}$
- The player bidding \$12 wins
- Player recieves: $\$300 + 3 \times \$288 = \$1,134$

Example (continued)

- Round 2 bids: $\{\$10, \$8, \$6\}$
- The player bidding \$10 wins
- Player receives: $\$300 + 1 \times \$300 + 2 \times \$290 = \$1,180$

- Round 3 bids: $\{\$8, \$6\}$
- The player bidding \$8 wins
- Player receives: $\$300 + 2 \times \$300 + 1 \times \$292 = \$1,192$

- Round 4 bids: $\{\$0\}$
- The player bidding \$0 wins
- Player receives: $\$300 + 3 \times \$300 = \$1,200$

Table: Net Cash Flow

Bidder/Round	Banker	1	2	3	4	Final
Banker	$1,200	−$300	−$300	−$300	−$300	$0
1	−300	1,164	−300	−300	−300	−36
2	−300	−288	1,180	−300	−300	−8
3	−300	−288	−290	1,192	−300	14
4	−300	−288	−290	−292	1,200	30

Trade-Offs

Winning early (net borrowers):

- should mean the participant has high-valued investment opportunity;
- allows capital to be raised quickly and project to be started;
- requires other participants pay you a discounted deposit and that you pay them a full deposit in all remaining rounds.

Winning late (net lenders):

- likely those with lower-valued rates-of-return;
- project money not obtained until late in hụi;
- one pays participants a discounted deposit and previous winners pay a full deposit.

Other Notes

- Under certain conditions, the hụi guarantees an efficient allocation of the scarce capital available to the coöperative.
- The hụi obviously facilitates inter-temporal smoothing, and appears to be implementable under primitive market conditions, such as those present in developing countries.
- An average hụi in Melbourne has around forty participants, each depositing as much as $500, which implies loans on the order of $20,000 for three to four years.

ROSCAs

- The hụi that we study is a special case of a class of institutions referred to in the literature as *Rotating Savings and Credit Associations*.
- Two varieties: *household* ROSCA and *business* ROSCA.

In a ROSCA, the participant receiving loan in a given period determined by

- bidding;
- lottery ("dice-shaking");
- fixed rotation order.

Road Map of Talk

1. Motivation and Introduction
2. Theoretical Models
3. Examples
4. Field Data
5. Econometric Model
6. Policy Experiments
7. Summary and Conclusions

Basic Set-Up

- Consider a set $\{0, 1, 2, \ldots, N\}$ of $(N+1)$ players: the banker plus N potential borrowers and lenders.
- At the beginning of the hụi-auction game, each participant deposits u with player 0, the banker.
- Each participant $n = 1, 2, \ldots, N$ receives an independent random draw R from $F_R^0(r)$ that has support $[\underline{r}, \bar{r}]$, with $f_R^0(r)$ that is strictly positive on $[\underline{r}, \bar{r}]$.
- Participant n's draw r_n is private information in the sense that each participant knows his draw, but not those of his opponents—only that opponents draws are independent and from the same distribution $F_R^0(\cdot)$.

Different Assumptions concerning Information

No alternative ways for these participants to borrow or to lend, which implies no constant rate of time preferences.

1. Initially, we assume that the rates-of-return for the hụi are drawn just once, in the initial period, when the total sum $(N \times u)$ is deposited.
2. In second model, we assume that the the rates-of-return for the hụi are redrawn in each period.
3. A third alternative might involve assuming that the rates-of-return for the hụi are drawn once at the beginning, but hit with a series of shocks as time proceeds.

First Model

- In each round of the hụi, a first-price, sealed-bid auction is conducted to decide who will win that round of the hụi and what discount will be paid, after which only the winning bid is revealed.
- For an hụi having N rounds, we introduce the following notation to denote the ordered rates-of-return of participants, from largest to smallest:

$$r_{(1)} \geq r_{(2)} \geq \cdots \geq r_{(N)}$$

and

$$w_1, \; w_2, \; \cdots \;, \; w_{N-1}, \; w_N = 0$$

to denote the winning bids in the N rounds of the hụi.

Implication of Winner for Information Release

- Participants will exit the hụi acccording to their rate-of-return draws—the highest first, then the second-highest, and so forth.
- In the first round, $r_{(1)}$ is

$$r_{(1)} = \sigma_1^{-1}(w_1)$$

  where w_1 is the winning bid in the first round, while $\sigma_1(\cdot)$ is the symmetric Bayes–Nash equilibrium bid function for first round.
- Conditional on having "observed" the highest-valued draw, the remaining rate-of-return draws are independent as well as identically distributed.

Using Dynamic Programming to Solve the Sequential Decision Problem

- We model the bid functions as a solution to a dynamic program.
- Two state variables: t and r.
- We seek to construct a sequence of optimal policy (equilibrium bid) functions $\{\sigma_t\}_{t=1}^{N}$.
- In round t, the optimal policy function σ_t maps the rate-of-return state R into the real line.

Value Function

$$V(r,t) = \max_{<s>} \left[tu + (N-t)(u-s) - u \sum_{i=1}^{N-t} \frac{1}{(1+r)^i} \right] \Pr\left(\text{win} \middle| s, w_1, w_2, \ldots, w_{t-1}\right)$$

$$+ \text{Discounted Expected Continuation Value.}$$

Proceeding Recursively

- Let's construct the policy functions and the value function recursively.
- In the last round, the optimal policy function, for all feasible R, is

$$\sigma_N(r) = 0.$$

- Hence, in the last round, N, for any feasible value of R,

$$V^*(r, N) = Nu.$$

- Let's move on to the second-to-last round.

Updating Information

- A representative participant in the second-to-last round who has rate-of-return r and faces only one other opponent.
- Suppose players use a monotonically increasing function $\hat{\sigma}_{N-1}(r)$:

$$\Pr\left(\text{win}\middle|s, w_1, w_2, \ldots, w_{N-2}\right) = \frac{F_R^0\left[\hat{\sigma}_{N-1}^{-1}(s)\right]}{F_R^0\left[r_{(N-2)}\right]} \equiv G_R^{N-2}\left[\hat{\sigma}_{N-1}^{-1}(s)\right]$$

where $g_R^{N-2}(\cdot)$ on $[\underline{r}, r_{(N-2)}]$.

Some Additional Manipulation

- We can now write

$$V(r, N-1) =$$

$$\max_{<s>} \left[(N-1)u + (u-s) - \frac{u}{(1+r)} \right] G_R^{N-2} \left[\hat{\sigma}_{N-1}^{-1}(s) \right] +$$

Discounted Expected Continuation Value.

- One part of the "Discounted Expected Continuation Value" is

$$\frac{V^*(r, N)}{(1+r)} = \frac{Nu}{(1+r)}.$$

- Another part is $(W_{N-1} - u)$, the net amount received from the winner of round $(N-1)$.

Recursive Solution

- Integrate out W_{N-1} for all values above the tendered bid s

$$V(r, N-1) =$$

$$\max_{<s>} \left[(N-1)u + (u-s) - \frac{u}{(1+r)} \right] G_R^{N-2}\left[\hat{\sigma}_{N-1}^{-1}(s)\right] +$$

$$\int_{\hat{\sigma}_{N-1}^{-1}(s)}^{r_{(N-2)}} \left(\left[\hat{\sigma}_{N-1}(x) - u\right] + \frac{Nu}{(1+r)} \right) g_R^{N-2}(x)\, dx.$$

Equilibrium Ordinary Differential Equation

- The first-order condition consistent with a Bayes–Nash equilibrium is an ordinary differential equation, which has known solution; for example,

$$\begin{aligned}
\sigma_{N-1}(r) &= \frac{\int_{\underline{r}}^{r} \left[\frac{x(N+1)u}{(1+x)}\right] F_R^0(x) f_R^0(x)\, \mathrm{d}x}{F_R^0(r)^2} + \underline{r}u \\
&= \left[\frac{\int_{\underline{r}}^{r} \left[\frac{x(N+1)}{(1+x)}\right] F_R^0(x) f_R^0(x)\, \mathrm{d}x}{F_R^0(r)^2} + \underline{r}\right] u \\
&\equiv \sigma_{N-1,1}(r) u.
\end{aligned}$$

- For later rounds, the corresponding differential equation must be solved numerically.

Homogeneity of $V^*(r, N-1)$ in u

$$V^*(r, N-1) = \left[Nu - \sigma_{N-1}(r) - \frac{u}{(1+r)} \right] G_R^{N-2}(r) +$$
$$\int_r^{r_{(N-2)}} \left(\left[\sigma_{N-1}(x) - u \right] + \frac{Nu}{(1+r)} \right) g_R^{N-2}(x)\, \mathrm{d}x,$$

which is homogeneous of degree one in u, too, so one can also construct a "unit" optimal valuation function $V_1^*(t, N-1)$, which simplifies the empirical analysis as well.

Unit Value Function in Round t

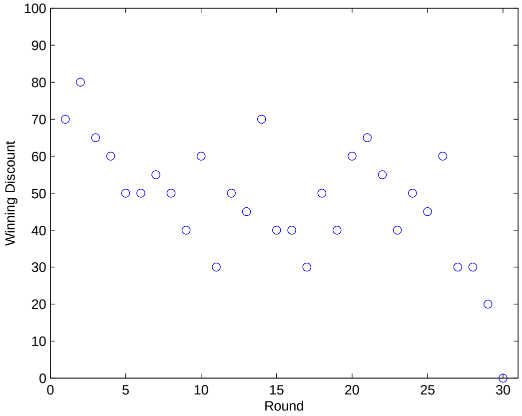
One can now proceed recursively: consider the next (previous) round, and focus on stationary point, take the the first-order conditions, solve the differential equation, and so forth.

$$V_1^*(r,t) = \left((N+1) - \frac{(1+r)}{r}\left[1 - \frac{1}{(1+r)^{N-t+1}}\right] - (N-t)\sigma_{t,1}(r)\right) G_R^{t-1}(r)^{N-t} +$$
$$\int_r^{r(t-1)} \left(\left[\sigma_{t,1}(x) - u\right] + \frac{V_1^*(r,t+1)}{(1+r)}\right)(N-t)G_R^{t-1}(x)^{N-t-1} g_R^{t-1}(x)\,\mathrm{d}x.$$

Properties of the Equilibrium

- Homogeneity carries through, so the calculations can be done in terms of a unit bid and unit value functions $\sigma_{t,1}(r)$ and $V_1^*(r,t)$, and then just multiplied u to get $\sigma_t(r)$ and $V^*(r,t)$, respectively.
- An efficient allocation of scare capital available to coöperative.
- Winning bids cannot rise across successive rounds of the hụi because the participants exit in order of rate-of-return, from highest to smallest, and the number of opponents fall as the hụi proceeds.

Winning Discount versus Rounds: $N = 30$, $u = \$500$



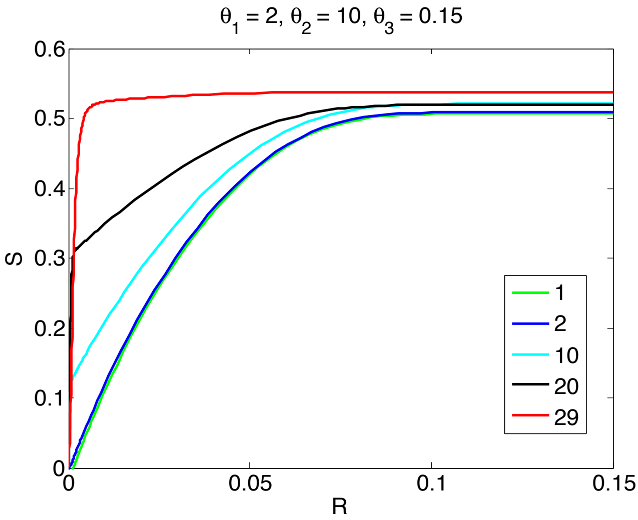
Second Model

- Remaining participants get new random draws from $F_R^0(r)$ in each round of the hụi.
- Under this assumption, building $V(r,t)$ recursively is much simpler than before.

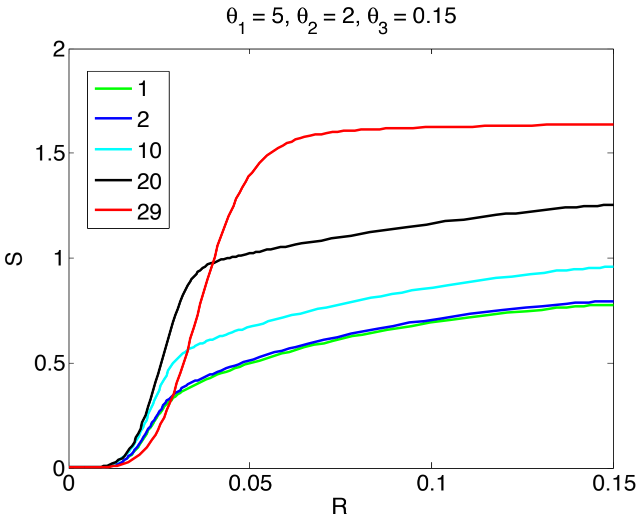
$$V(r,t) = \max_{<s>} \left[tu + (N-t)(u-s) - \sum_{i=1}^{N-t} \frac{u}{(1+r)^i} \right] F_R^0\left[\hat{\sigma}_t^{-1}(s)\right]^{N-t} + \int_{\hat{\sigma}_t^{-1}(s)}^{\bar{r}} \left(\left[\hat{\sigma}_t(x) - u\right] + \mathcal{E}\left[\frac{V^*(R,t+1)}{(1+R)}\right] \right) (N-t) F_R^0(x)^{N-t-1} f_R^0(x)\, \mathrm{d}x.$$

- Here, increases in winning discount bid across rounds of hụi can obtain.

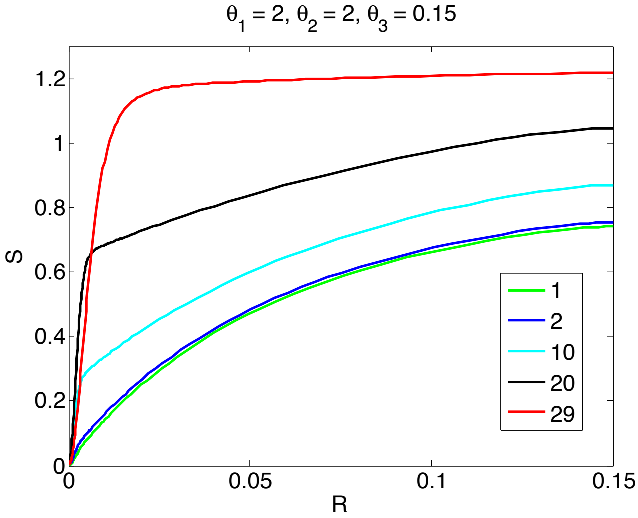
Equilibrium Bid Functions: $N = 30$, $u = 1$



Equilibrium Bid Functions: $N = 30$, $u = 1$



Equilibrium Bid Functions: $N = 30$, $u = 1$



Field Data

- A former hụi banker, now retired, has graciously provided us with a small sample of bids from 22 hụi, which were held in the early 2000s in a suburb of Melbourne, Australia.
- As part of our agreement with this banker, we can say very little more than this.
- Specifically, we cannot provide demographic characteristics of the participants, nor can we describe the activities in which the funds from the hụi were invested.
- We can, however, describe the important economic variables for the sample of hụi.

Table: Sample Descriptive Statistics

Variable	Sample Size	Mean	St.Dev.	Minimum	Median	Maximum
u_h	22	345.45	126.22	200	300	500
N_h	22	35.95	11.00	21	36	51
w_{ht}	769	58.98	11.00	5	40	150
$w_{ht,1}$	769	0.1579	0.0764	0.0200	0.1500	0.4500

Econometric Model

- We have demonstrated that this model is identified non-parametrically, at least in the last round of the hụi.
- Introduce $G_S^t(s)$ and $g_S^t(s)$ to denote the distribution of equilibrium bids in round t of an hụi having N rounds with deposit u.
- Denote by $s_{t,1}$ the unit bid in round t of an hụi having deposit u; in other words, $s_{t,1}$ is (s_t/u).
- Denote by $G_{S,1}^t(\cdot)$ and $g_{S,1}^t(\cdot)$ the cumulative distribution and probability density functions of equilibrium unit bids in round t.

In round t, Bayes–Nash equilibrium is defined by the following differential equation:

$$\frac{\mathrm{d}\sigma_t(r)}{\mathrm{d}r} = \Bigg((N+1)u - u\frac{(1+r)}{r}\left[1 - \frac{1}{(1+r)^{N-t+1}}\right] - \frac{V^*(r,t+1)}{(1+r)} - (N-t+1)\sigma_t(r)\Bigg)\frac{f_R^0(r)}{F_R^0(r)}$$

One can undertake the change of random variables proposed by Guerre *et al.* when constructing the inverse bid function.

$$1 = \Bigg((N+1)u - u\frac{(1+r)}{r}\left[1 - \frac{1}{(1+r)^{N-t+1}}\right] - \frac{V^*(r,t+1)}{(1+r)} - (N-t+1)\sigma_t(r)\Bigg)\frac{g_S^t(s_t)}{G_S^t(s_t)}$$

Can express using unit bid and value functions

$$\underbrace{(N+1)-(N-t+1)s_{t,1}-\frac{G^t_{S,1}(s_{t,1})}{g^t_{S,1}(s_{t,1})}}_{\text{observed}}=\left(\underbrace{\frac{1+r}{r}\left[1-\frac{1}{(1+r)^{N-t+1}}\right]}_{\text{known fcn of } r}+\underbrace{\frac{V^*_1(r,t+1)}{(1+r)}}_{\text{unknown fcn of } R}\right)$$

Unknown function depends on $F^0_R(\cdot)$ except in one case:

$$V^*_1(r,N)=N.$$

- We really do not have enough data (22 observations) to implement a serious non-parametric analysis.
- In light of this dearth of data, in order to implement our theoretical model, we have been forced to make a parametric assumption concerning the distribution of R.

- Consider $\{(u_h, N_h, w_{1,h}, w_{2,h}, \ldots, w_{N_h-1})\}_{h=1}^{H}$, a sample of H hụi, indexed by $h = 1, 2, \ldots, H$.
- Under our second informational assumption,

$$w_{t,h} = \sigma_t\Big[r_{(1:N_h-t+1)}; u_h, N_h\Big]$$

where we have now made explicit the dependence of the winning bid discounts on both u_h and N_h.

- Denote the cumulative distribution function of $R_{(1:N_h-t+1)}$ for participants at hụi h by

$$F_{(1:N_h-t+1)}(r; \boldsymbol{\theta}, N_h, t) = (N_h - t + 1) \int_0^{F_R^0(r;\boldsymbol{\theta})} x^{N_h-t}\, \mathrm{d}x$$

and its probability density function by

$$f_{(1:N_h-t+1)}(r; \boldsymbol{\theta}, N_h, t) = (N_h - t + 1) F_R^0(r; \boldsymbol{\theta})^{N_h-t} f_R^0(r; \boldsymbol{\theta}).$$

- Now, the probability density function of the winning bid in round t of hụi h is then

$$f_{W_{t,h}}(w;\boldsymbol{\theta},u_h,N_h,t) = \frac{f_{(1:N_h-t+1)}\left[\sigma_t^{-1}(w;\boldsymbol{\theta},u_h,N_h);\boldsymbol{\theta},N_h,t\right]}{\sigma_t'\left[\sigma_t^{-1}(w;\boldsymbol{\theta},u_h,N_h);,\boldsymbol{\theta},u_h,N_h,t\right]}.$$

- Thus, collecting the u_hs in the vector $\boldsymbol{u}$, the N_h in the vector $\boldsymbol{N}$, and the $w_{t,h}$s in the vector $\boldsymbol{w}$, the logarithm of the likelihood function can be written as

$$\mathcal{L}(\boldsymbol{\theta};\boldsymbol{u},\boldsymbol{N},\boldsymbol{w}) = \sum_{h=1}^{H}\sum_{t=1}^{N_h-1}\Bigg[\log\left(f_{(1:N_h-t+1)}\left[\sigma_t^{-1}(w_{t,h};\boldsymbol{\theta},u_h,N_h);\boldsymbol{\theta},N_h,t\right]\right)- \log\left(\sigma_t'\left[\sigma_t^{-1}(w_{t,h};\boldsymbol{\theta},u_h,N_h);\boldsymbol{\theta},u_h,N_h,t\right]\right)\Bigg].$$

To estimate this empirical specification:

0. set $k = 0$ and initialize $\boldsymbol{\theta}$ at $\tilde{\boldsymbol{\theta}}^k$;
1. solve for $\tilde{\sigma}^k_{t,1}(r) = \sigma_{t,1}(r; N_h, \tilde{\boldsymbol{\theta}}^k)$ and $\tilde{V}^k_1(r,t)$ for $t = 1, 2, \ldots, N_h - 1$ and $h = 1, 2, \ldots, H$;
2. for each $w_{t,h}$ in $\boldsymbol{w}$, then solve $(w_{t,h}/u_h) = \tilde{\sigma}^k_{t,1}\left[\tilde{r}^k_{(1:N_h-t)}\right]$—viz., find the $\tilde{r}^k_{(1:N_h-t)}$ consistent with $\tilde{\boldsymbol{\theta}}^k$;
3. form the logarithm of the likelihood function for iteration k and maximize it with respect to $\boldsymbol{\theta}$, taking into account the following constraints:

$$\frac{w_{t,h}}{u_h} \leq \tilde{\sigma}^k_{t,1}(\bar{r}) \quad t = 1, 2, \ldots, N_h - 1;\ h = 1, 2, \ldots, H;$$

4. check for an improvement in the objective function: if no improvement obtains, then stop, otherwise increment k and update $\tilde{\boldsymbol{\theta}}^k$ and return to step 1.

Policy Experiments

- Once we have some empirical estimates, we can evaluate alternative instituions; for example, consider
  - a Vickrey auction;
  - a lottery.
- We can also analyze the extent (importance) of inefficiencies in the hụi.
- For although the hụi is efficient in each round, it is unclear whether it is dynamically efficient.

Summary and Conclusions

- Using the theory of non-cooperative games under incomplete information, we have analyzed the hụi—a borrowing and lending institution used by Vietnamese immigrants in Australia and New Zealand, in particular, but in other parts of the world as well.
- Essentially, the hụi is a sequential, double auction among the participants in a collective.
- Within the symmetric independent private-values paradigm, we constructed the Bayes–Nash equilibrium of a sequential, first-price, sealed-bid auction game and then investigated the properties of the equilibrium using numerical methods.

- We also demonstrated that this model is non-parametrically identified, at least in the second-to-round of the hụi.
- Subsequently, we used this structure to interpret field data gathered from a sample of hụi held in Melbourne, Australia during the early 2000s.
- We also plan to investigate two simple policy experiments— one involving a shift to a second-price, sealed-bid format and the other a shift to a lottery, which is how a mechanism like the hụi is implemented in Mexico.
- While it is obvious that the hụi will do much better in allocating capital efficiently than the random allocation, our estimates will provide some notion of the efficiency gain from using the hụi.